安徽理工大学"基础物理研究中心"成立大会暨学术报告会
安徽 淮南, 2024.05.22-05.25

r-过程研究中原子核质量与β衰变寿命的理论描述

牛中明

安徽大学 物理与光电工程学院

2024年05月23日

Outline

1. Introduction
2. Nuclear physics inputs
   - ⋆ Nuclear masses
   - ⋆ Nuclear β-decay half-lives
3. r-process simulations
4. Summary and perspectives

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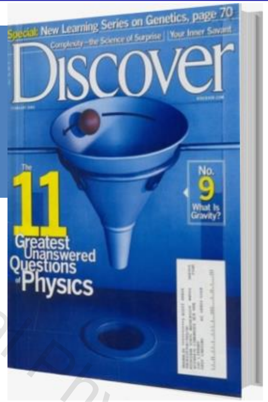
Introduction

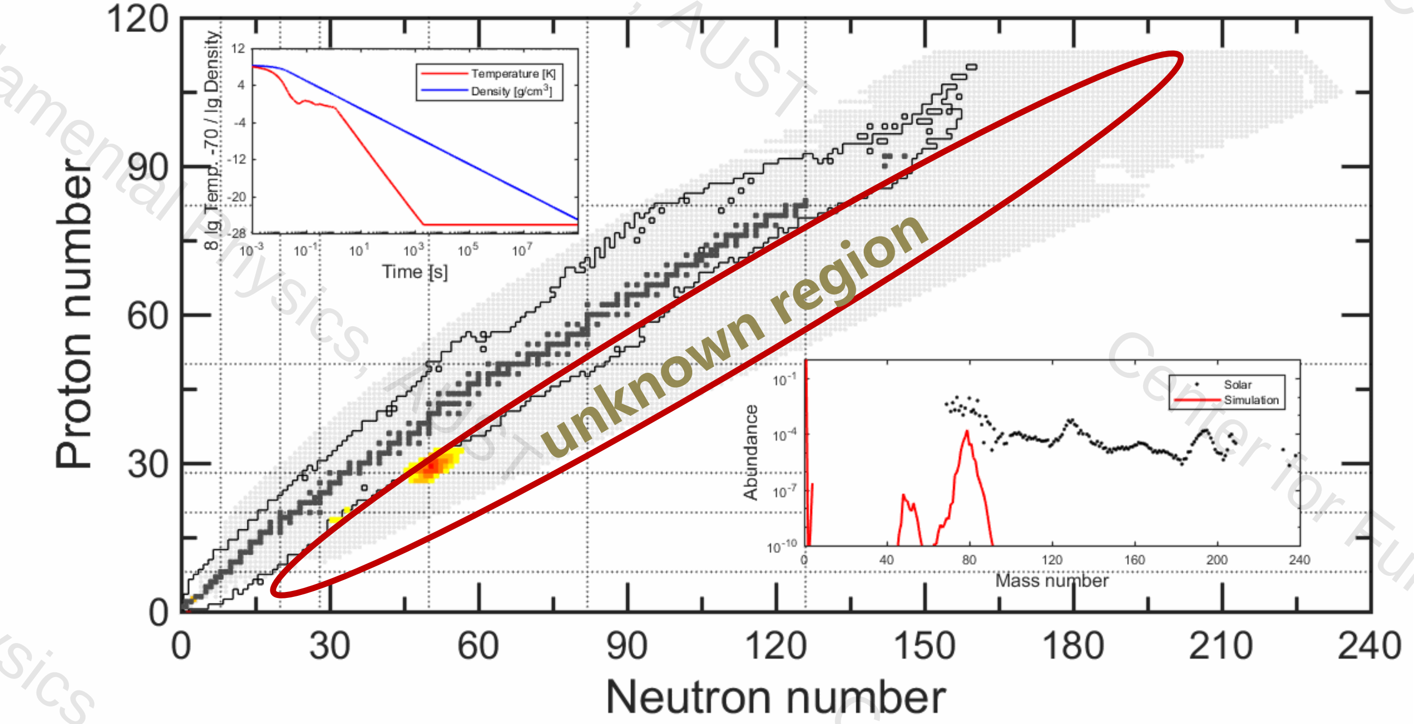
Origin of elements

How were the heavy elements from iron to uranium made?

DISCOVER:
The 11 greatest unanswered questions of Physics



- **Rapid neutron-capture process (r-process)**



Key nuclear physics inputs:

✓ **Nuclear mass ➔ r-process path**

✓ **β-decay half-life ➔ r-process time scale**

Accurate theoretical predictions of nuclear masses and β-decay half-lives are crucial to understanding the r-process.

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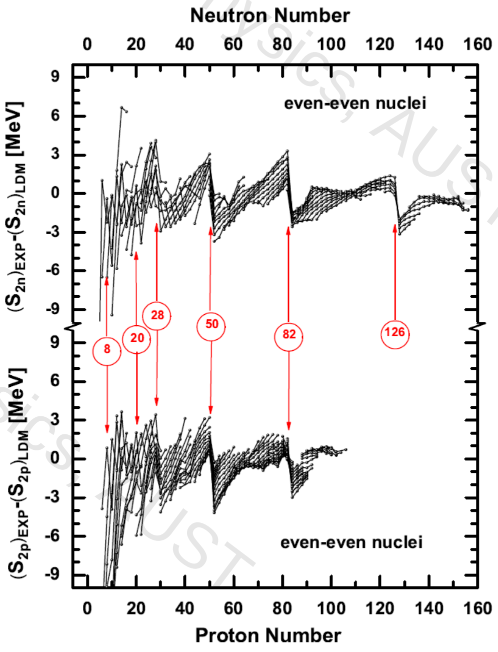
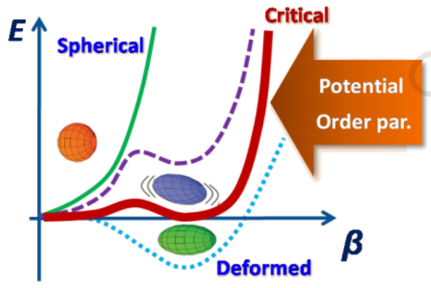
Nuclear masses

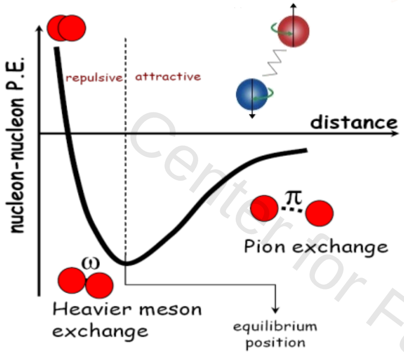
- Nuclear mass is a fundamental quantity in nuclear physics. It plays important roles not only in nuclear physics, but also in other branches of physics, such as astrophysics and nuclear engineering. [Lunney2003RMP, Burbidge1957RMP]

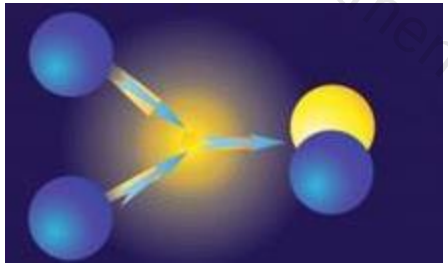
★ Nuclear physics: it contains wealth of nuclear structure information such as magic number and shape transition, and it is widely used to extract nuclear effective interactions.

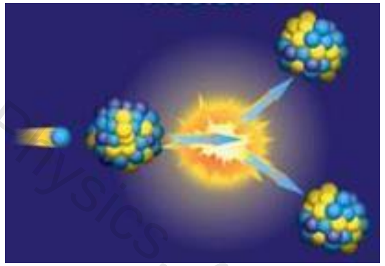
★ Other branches: it is essential to determine nuclear reaction and decay energies, so it is important in astrophysics and nuclear engineering.

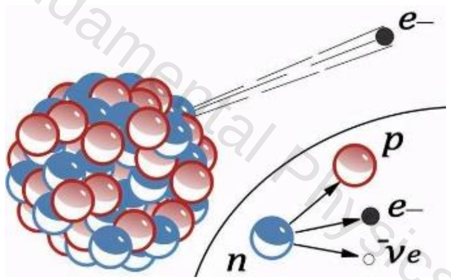


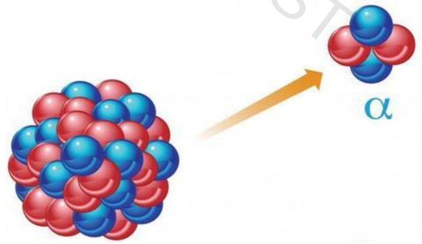




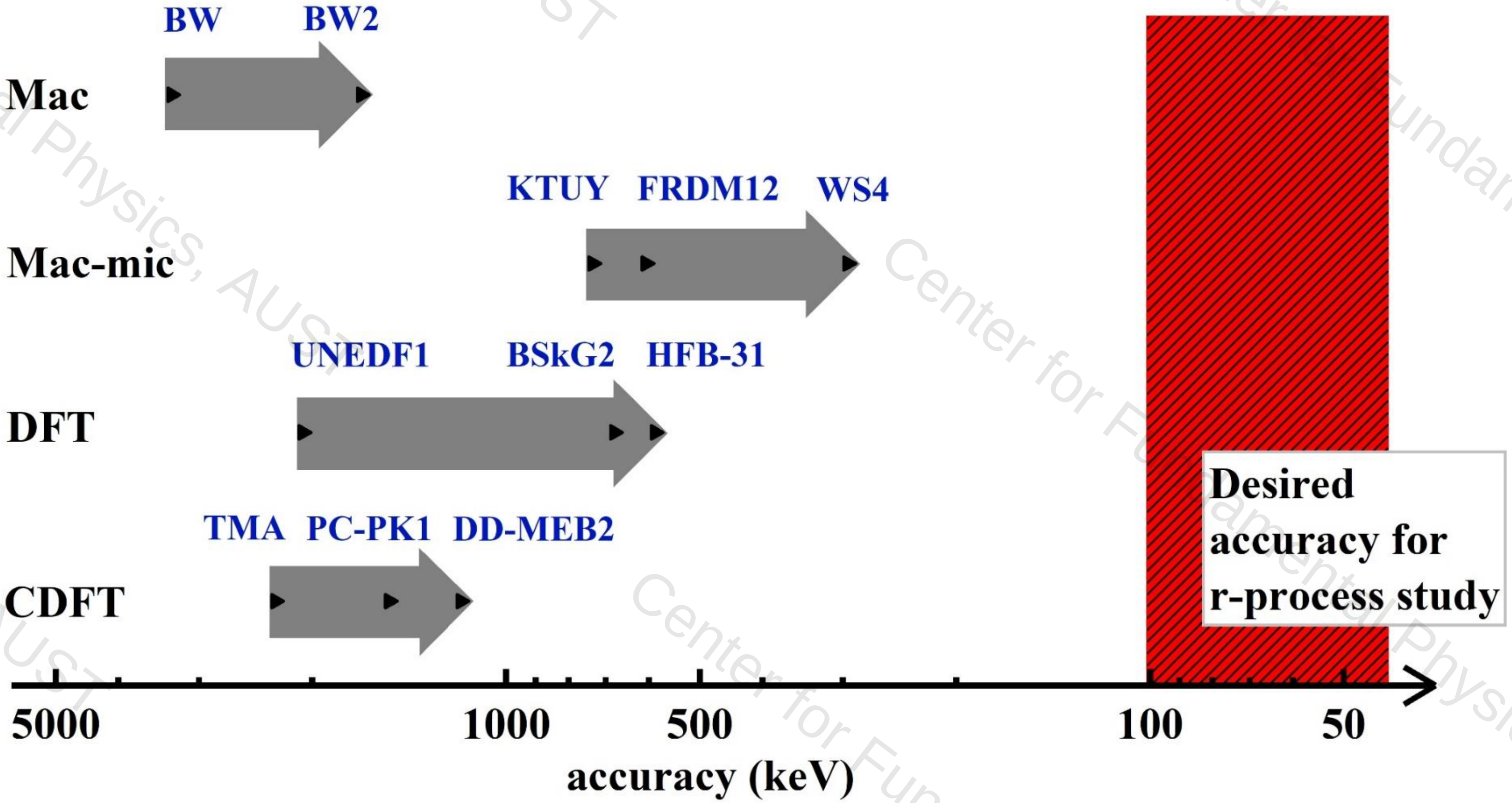






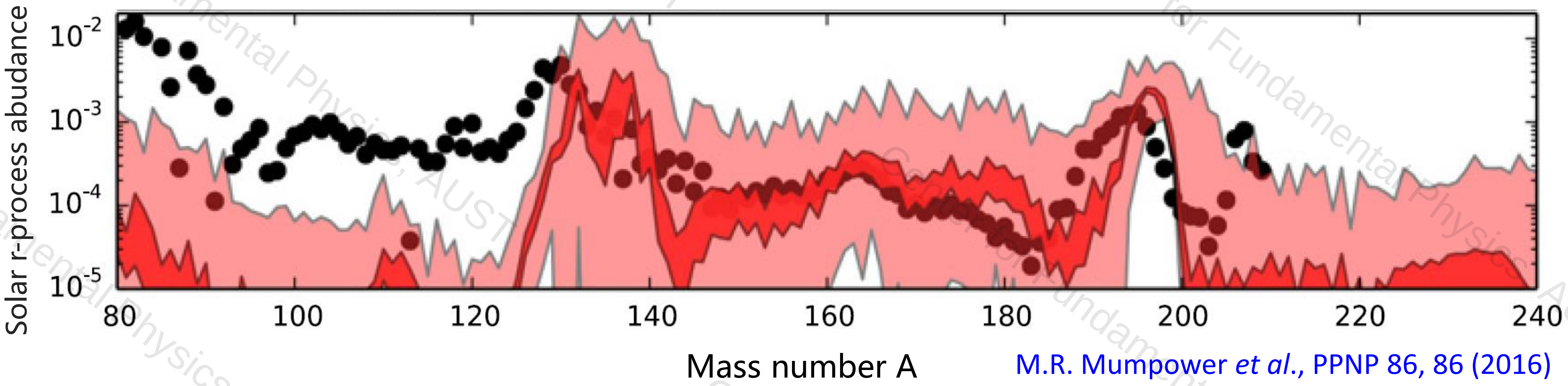


Nuclear mass models



- ⋆ Macroscopic mass models: BW, BW2 [Weizsäcker1935ZP, Bethe1937RMP, Kirson2008NPA]
- ⋆ Macro-microscopic mass models: KTUY, FRDM, WS4 [Koura2005PTP, Moller2012PRL, Wang2014PLB]
- ⋆ Microscopic DFT mass models: UNEDF1, BSkG2, HFB-31 [Kortelainen2012PRC, Ryssens2022EPJA, Goriely2016PRC]
- ⋆ Microscopic CDFT mass models: TMA, PC-PK1, DD-MEB2 [Geng2005PTP, Zhang2022ADNDT, Arteaga2016EPJA]

Influence of mass uncertainties

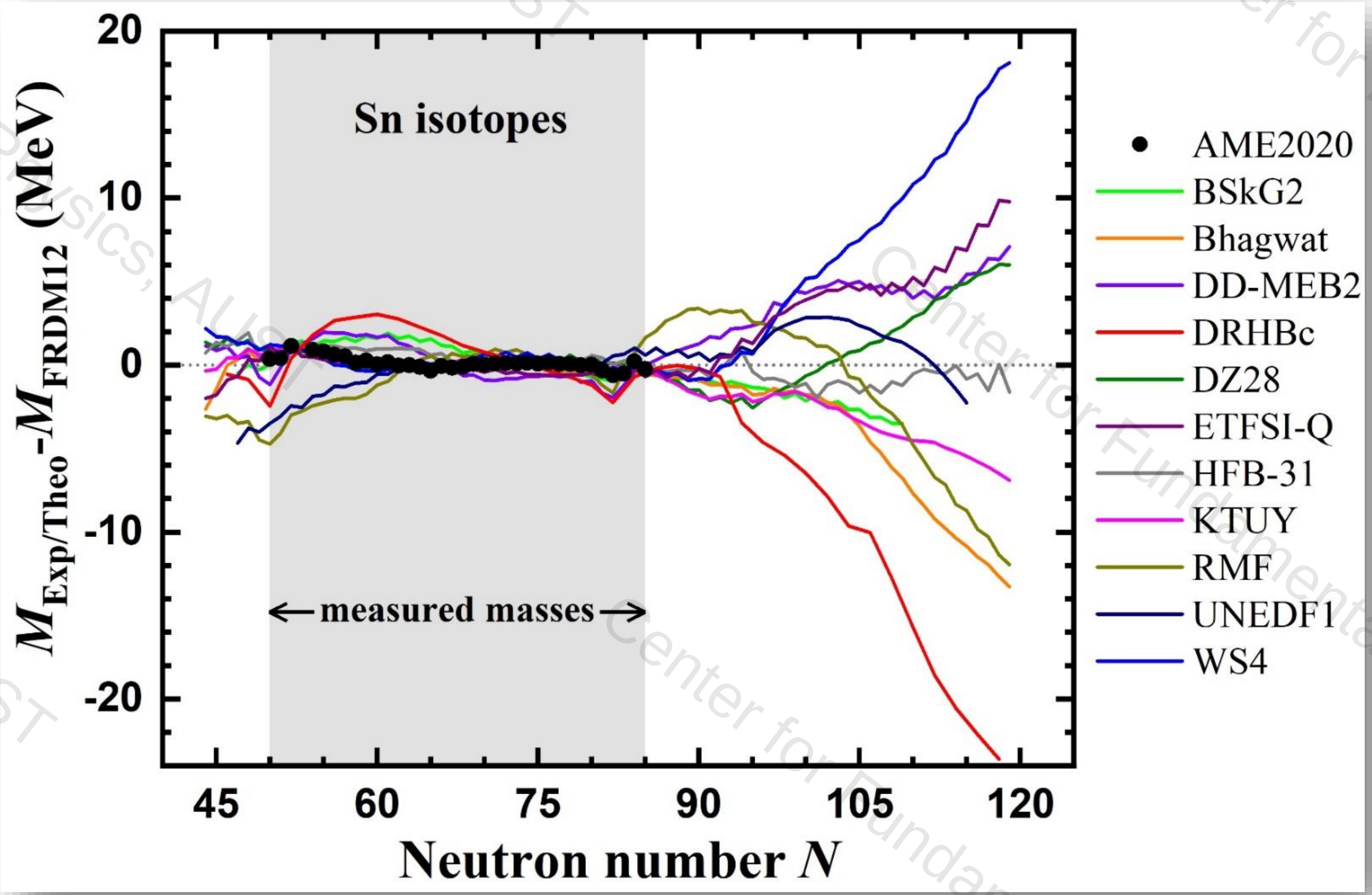


M.R. Mumpower *et al.*, PPNP 86, 86 (2016)

Lighter and **darker** shaded bands represent the influence of mass uncertainties of **500 keV** and **100 keV** to r-process abundances, respectively.

Accurate description of r-process abundance requires nuclear mass prediction accuracy within 100 keV.

Mass deviations from different models



★ Different nuclear mass models with comparable accuracy in the known region can extrapolate quite differently out to the neutron drip line.

Macroscopic mass models: semi-empirical formulas

$$\text{BW:}\quad B = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_{sym}\frac{(N-Z)^2}{4A} + a_p \frac{\delta}{\sqrt{A}}$$

$$\text{BWK:}\quad B = a_v A - a_s A^{2/3} + a_r A^{1/3} - a_c \frac{Z^2}{A^{1/3}} + a_{xc}\frac{Z^{4/3}}{A^{1/3}} - a_{sym}\frac{(N-Z)^2}{4A} + a_{st}\frac{(N-Z)^2}{A^{4/3}} - a_w \frac{|N-Z|}{A} + a_p \frac{\delta}{\sqrt{A}} + \alpha_m P + \beta_m P^2$$

$$\text{BWN:}\quad B = a_v A - a_s A^{2/3} + a_r A^{1/3} - a_c \frac{Z^2}{A^{1/3}} + a_{xc}\frac{Z^{4/3}}{A^{1/3}} - a_{symI} I^2 A + a_p \frac{\delta_{np}}{A^{1/3}} + \alpha_m P + \beta_m P^2$$

$$+ a_{sh}(\nu_p + \nu_n) + b_{sh}\delta_{sh} e^{c_{sh}(\nu_p^2 + \nu_n^2)}$$

$$a_{symI} = c_{sym}\left(1 - \frac{k}{A^{1/3}} + \frac{2-|I|}{2+|I|A}\right) \xrightarrow[|I|A \gg 2 \gg |I|]{I=(N-Z)/A} a_{symI} I^2 A = c_{sym}\frac{(N-Z)^2}{A} - c_{sym} k \frac{(N-Z)^2}{A^{4/3}} + 2c_{sym}\frac{|N-Z|}{A}$$

$$\delta_{shell} = \begin{cases} -1 & 8 \le Z \le 24 \text{ and } 8 \le N \le 24 \\ 0 & 24 < Z \le 39 \text{ and } 8 \le N \le 66 \\ 0 & 8 \le Z \le 24 \text{ and } 24 < N \le 66 \\ 1 & \text{else} \end{cases}$$

$$\delta_{np} = \begin{cases} 2-|I| & N \text{ and } Z \text{ even} \\ |I| & N \text{ and } Z \text{ odd} \\ 1-|I| & N \text{ even, } Z \text{ odd, and } N > Z \\ 1-|I| & N \text{ odd, } Z \text{ even, and } N < Z \\ 1 & N \text{ even, } Z \text{ odd, and } N < Z \\ 1 & N \text{ odd, } Z \text{ even, and } N > Z \end{cases}$$

★ The rms deviations with respect to the experimental masses in AME2020 (Z,N≥8) are reduced from 3.067 MeV of BW, 1.626 MeV (reduction: 46.98%) of BWK to 0.902 MeV (reduction: 44.53%) of BWN, which is **the first semi-empirical mass formula crossing the 1 MeV accuracy threshold**.

BW: C. F. Von Weizsacker, Z. Phys. 96, 431 (1935); H. A. Bethe *et al.*, Rev. Mod. Phys. 8, 82 (1936).
BWK: M. W. Kirson, Nucl. Phys. A 798, 29 (2008).

Macroscopic mass models: semi-empirical formulas

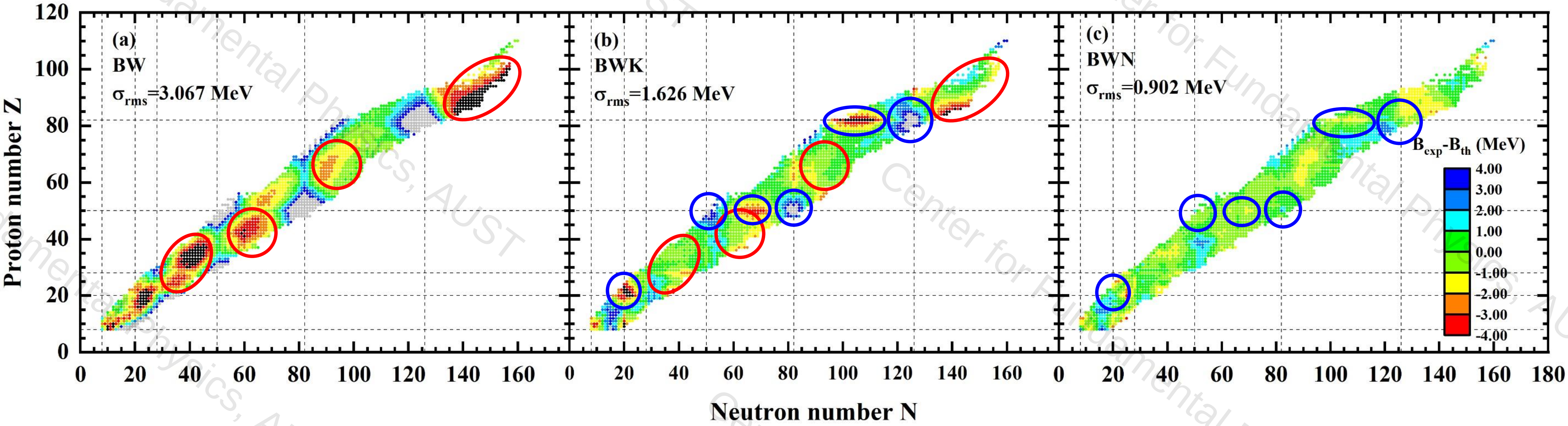


Fig: The difference between the experimental binding energies and the results of the semi-empirical formulas BW, BWK, and BWN, respectively. The black dashed line indicates the magic number.

★ BW2: the inclusion of the term $\alpha_m P + \beta_m P^2$ significantly improves the mass description of nuclei between magic number, which are generally deformed nuclei.

★ BWN: the inclusion of the shell correction term further significantly improves the mass description of nuclei near magic number, in which the shell effects are very important.

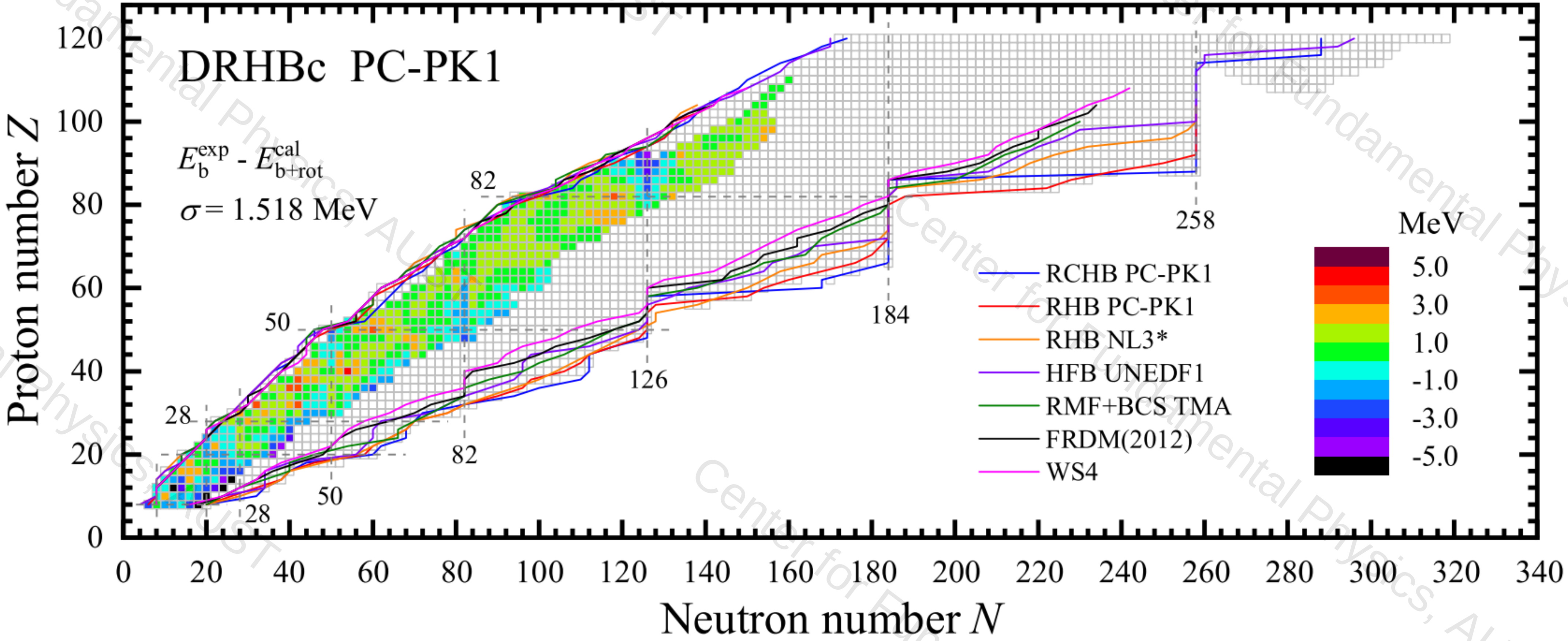
Results and discussion

CDFT mass model: The DRHBc Mass Table Collaboration

The DRHBc Mass Table Collaboration (31 Universities and Institutions from China, South Korea, and Japan): Nuclear mass table in deformed relativistic Hartree-Bogoliubov theory in continuum.

https://drhbctable.jcnp.org/collaboration.html

CDFT mass predictions



K. Zhang *et al*. (DRHBc Mass Table Collaboration), ADNDT 144, 101488 (2022)

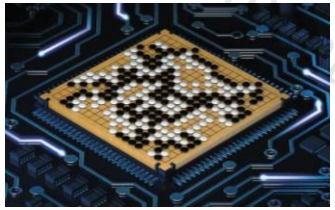
★ The σ_{rms} of DRHBc is 1.518 MeV, providing **one of the best microscopic descriptions for nuclear masses**.

★ The DRHBc calculations generally predict a more extended neutron drip line than other DF calculations, mainly due to the proper treatment of the continuum and the density functional adopted.

CDFT mass predictions

K. Zhang *et al*. (DRHBc Mass Table Collaboration), ADNDT 144, 101488 (2022)

Machine learning in nuclear mass predictions



game of go

computer vision

translation

autonomous car

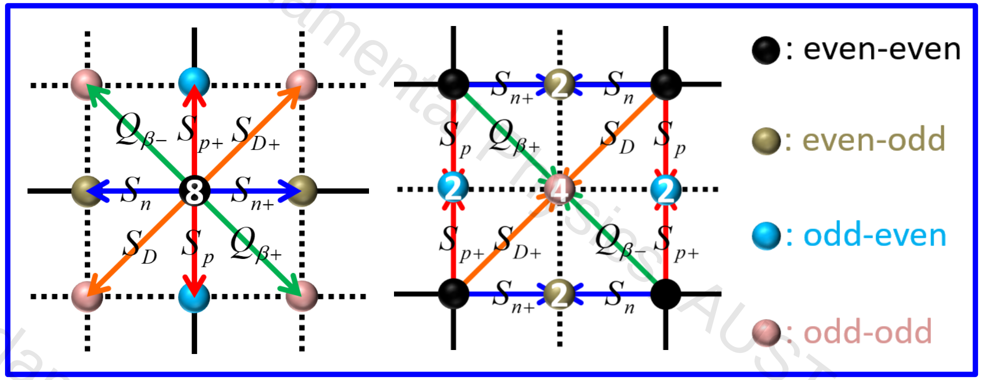
voice recognition

Machine learning in nuclear mass predictions

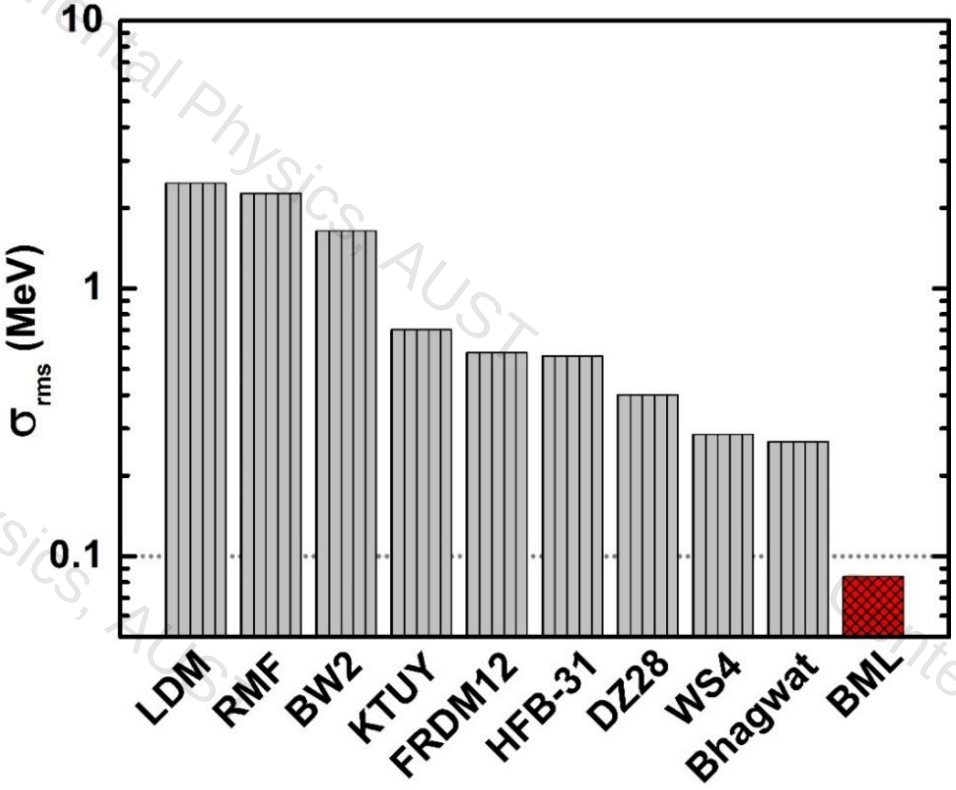
- ★ ANN: Gazula1992NPA, Athanassopoulos2004NPA, Bayram2014ANE, **Zhang2017JPG, Ming2022NST,** Yuksel2021IJMPE, **Li2022PRC**
- ★ BNN: Utama2016PRC, **Niu2018PLB, Niu2019PRC, Niu2022PRCL**, Rodriguez2019EPL, Rodriguez2019JPG
- ★ CNN: **Yang2023PRC**
- ★ LightGBM: **Gao2021NST**
- ★ KRR: **Wu2020PRC, Wu2021PLB**
- ★ NBP: **Liu2021PRC**
- ★ RBF: **Wang2011PRC, Niu2013,2016PRC,2018SciB**
- ★ BGP: Neufcourt2018,2020PRC, Neufcourt2019PRL
- ★ SVM: Clark2006IJMPB
- ★ CLEAN: Morales2010PRC
- ★ …….

Results and discussion

Bayesian Machine Learning (BML) mass model



Model	M	S_n	S_{2n}	S_p	S_{2p}	S_D	Q_β
FRDM12	0.576	0.340	0.442	0.341	0.420	0.411	0.450
HFB-31	0.559	0.451	0.456	0.489	0.496	0.566	0.557
WS4	0.285	0.254	0.261	0.261	0.300	0.324	0.327
BML	**0.084**	**0.078**	**0.105**	**0.083**	**0.111**	**0.096**	**0.099**



★ A nuclear mass model with accuracy smaller than 100 keV in the known region is constructed.

★ Its accuracies to S_x and Q_x are at least about 3 times higher than other mass models.

Z.M. Niu and H.Z. Liang, PRC 106, L021303 (2022)

Extrapolation of BML

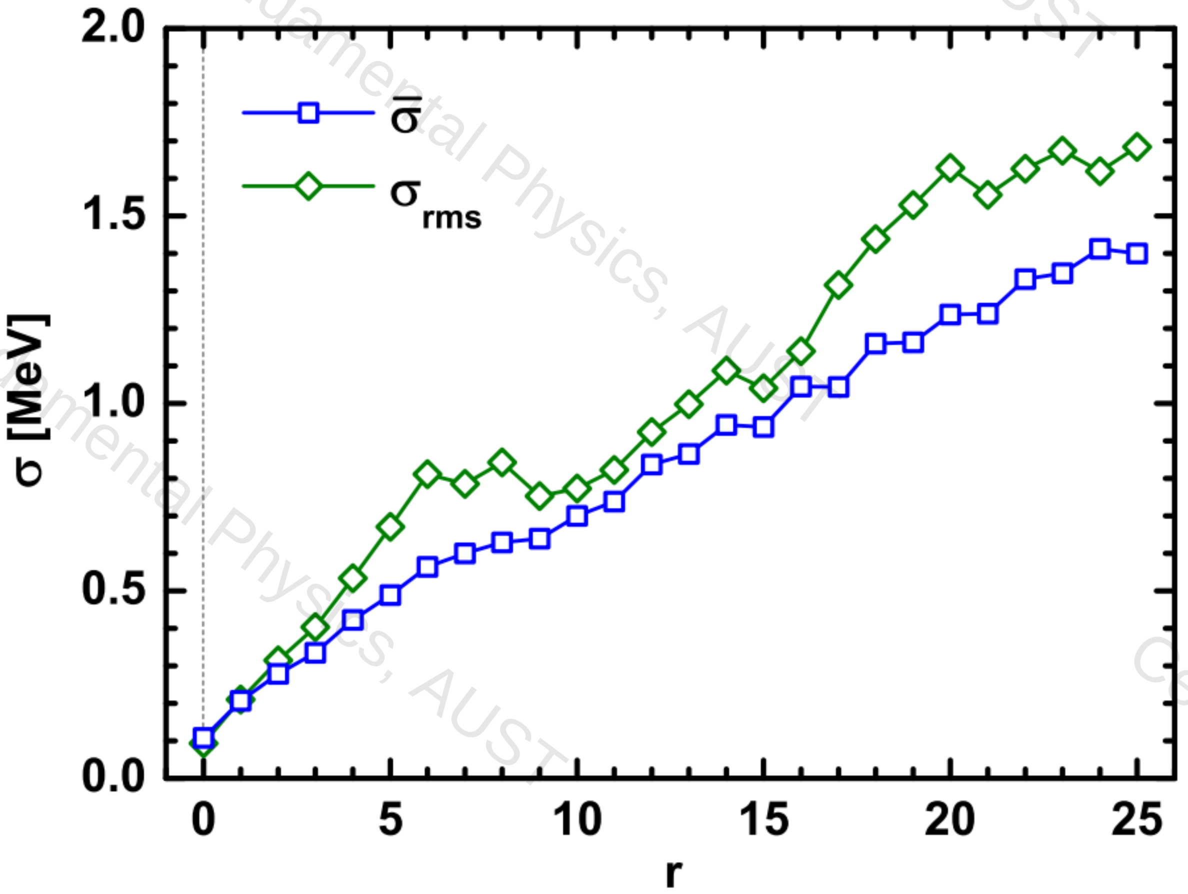


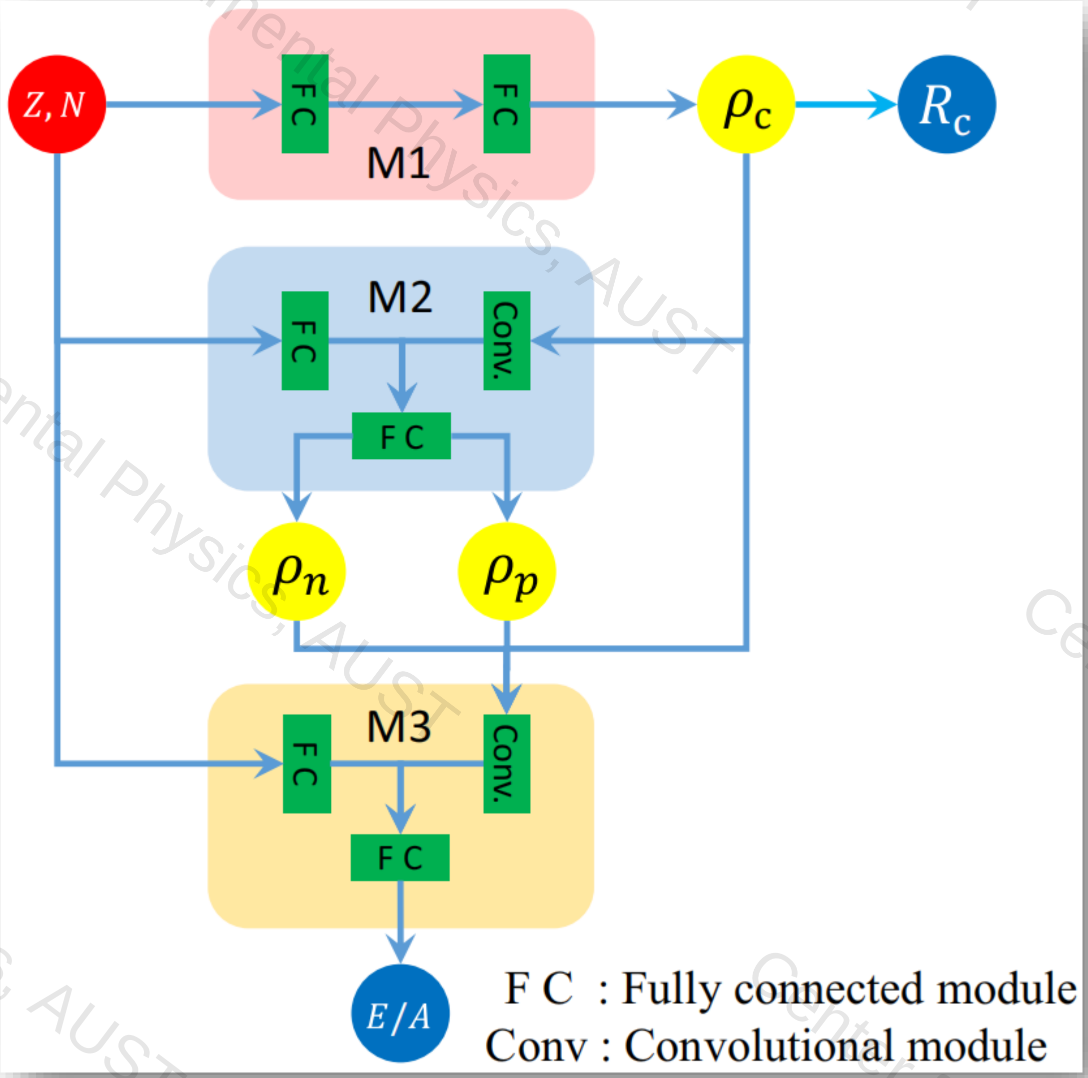
Fig: Average theoretical uncertainties $\bar{\sigma}$ (squares) of mass predictions by BML as a function of minimum distance r to the isotopes in the learning set. The rms deviations σ_{rms} of BML mass predictions with respect to the corresponding FRDM12 values are shown with diamonds.

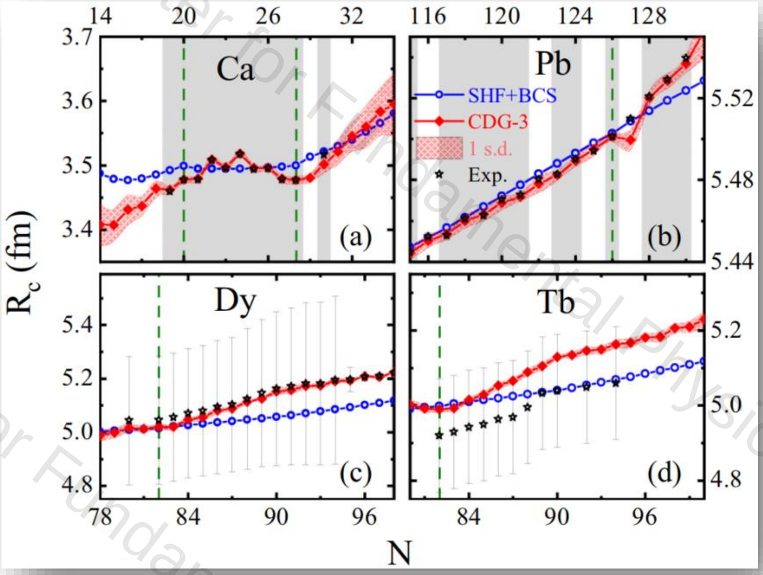
Taking the FRDM12 mass predictions as the Pseudoexperimental data:

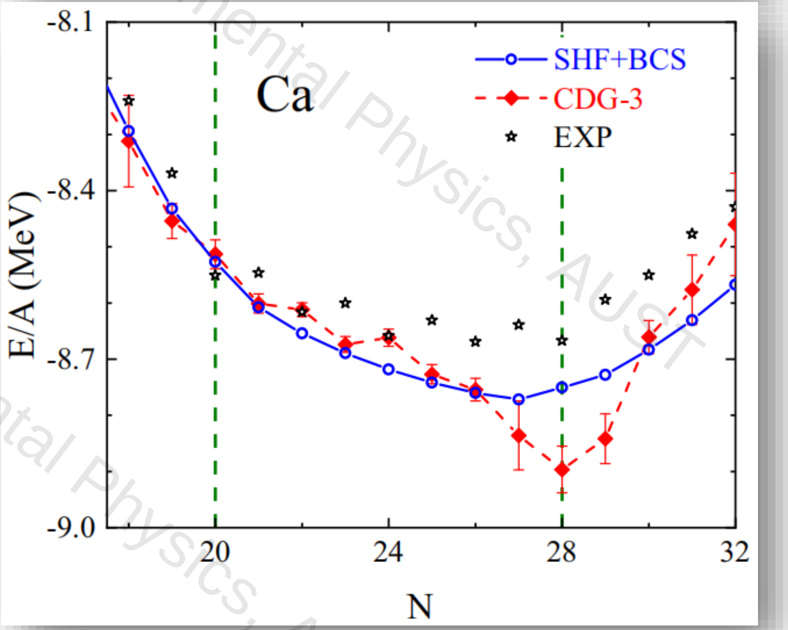
★ The BML model can well reproduce the Pseudoexperimental data within 100 keV for nuclei in the known region.

★ The rms deviation between BML predictions and Pseudoexperimental data increases as the increase of the distance *r*. It is very similar to the average error of BML, which indicates **the BML model could give reasonable evaluations of the theoretical uncertainties**.

Density functional: FNN+CNN



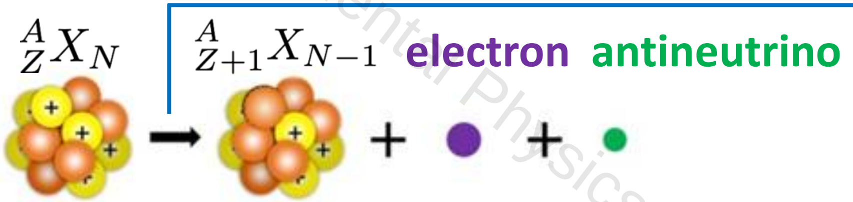


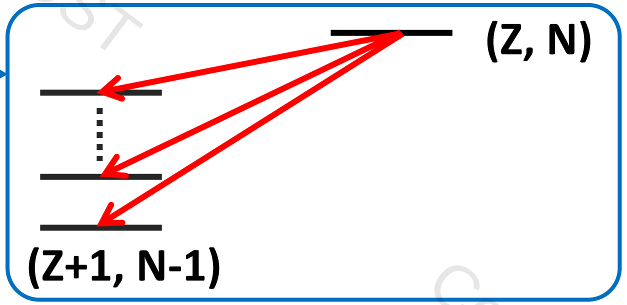


Z. X. Yang *et al.*, PRC 108, 034315 (2023)

Outline

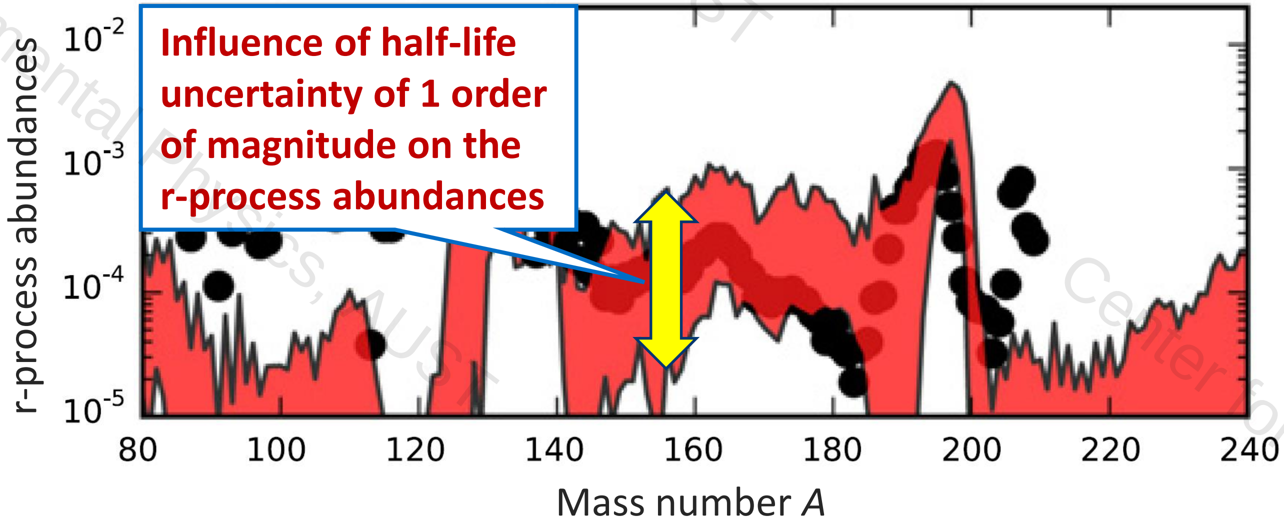
Nuclear β-decay



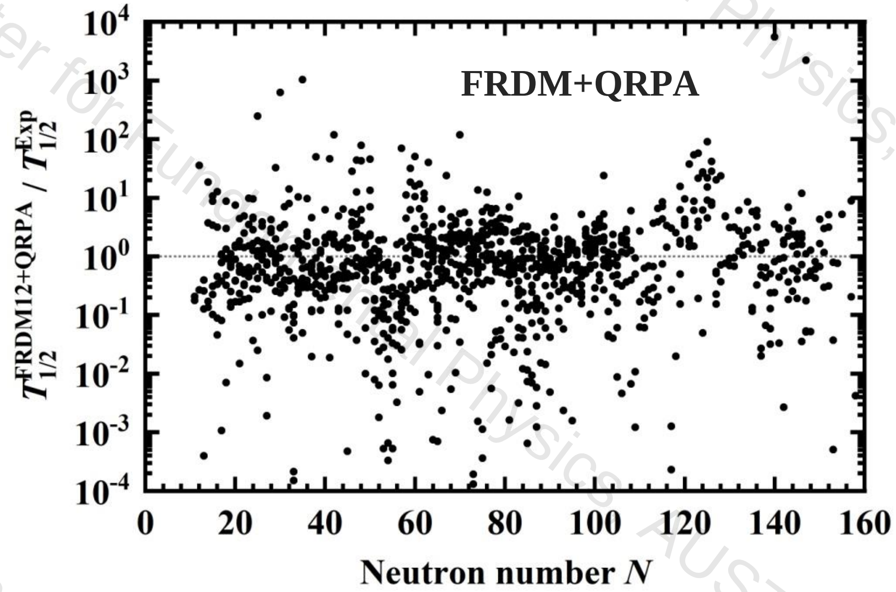


In allowed GT approximation:

$$T_{1/2} = \frac{D}{g_A^2 \sum_m B_{GT}(E_m) f(Z, A, E_m)}$$



Mumpower *et al.*, PPNP 86, 86 (2016)



Möller *et al.*, ADNDT 125, 1 (2019)

Constructing a nuclear model that accurately describes β-decay half-lives is critical to understanding the origin of heavy elements in the universe.

Nuclear models for β-decay half-lives

- Phenomenological formula Pfeiffer2000Report, **Zhang2006PRC,2007JPG**, **Zhou2017SCPMA**, **Xia2024APS**
- Gross theory Takahashi1969PTP, Tachibana1990PTP, Nakata1997NPA, Koura2017PRC, **Fang2022PRC**
- Shell model Pinedo1999PRL, Caurier2002PRC, Langanke2003RMP, **Zhi2013PRC**
- Quasiparticle random phase approximation (QRPA)
  - Nilsson BCS+QRPA: Staudt1990ADNDT, Hirsch1993ADNDT, Nabi1999ADNDT
  - FRDM+QRPA: Möller1997,2018ADNDT, Möller2003PRC
  - Woods-Saxon+QRPA: **Ni2012JPG**
  - SHF BCS+QRPA: Sarriguren2005, 2010, 2011PRC
  - DF(Fayans)+CQRPA: Borzov1996ZPA, Borzov2003,2005PRC, Borzov2008NPA
  - ETFSI(Skyrme)+CQRPA: Borzov1997NPA, Borzov2000PRC
  - SHF(BCS)+(Q)RPA: **Bai2010PRL**, Minato2013PRL, Minato2022PRC
  - SHFB+QRPA/FAM/QPVC: Engel1999PRC, **NiuYF2015PRL,2018PLB**, FAM: Ney2020PRC
  - RHB+QRPA: Nikšić2005PRC, Marketin2007,2016PRC, **Wang2016JPG**, **NiuZM2013PRC(R)**
  - RHFB+QRPA: **NiuZM2013PLB**
- Machine learning Costiris2009PRC, **Li2022SSPMA**, **Niu2019PRC**, **Li2024JPG**

Empirical formula of β-decay half-lives

- According to the Fermi theory of β-decay, neglecting the effect of transition strength, taking the fine structure constant α as a small quantity, and using the extreme relativistic limit approximation $E_e \rightarrow p_e c$, one obtains

Formula 1: $$\ln(T_{1/2}) = \ln(30 D m_e^5 c^{10}) + (\alpha^2 Z^2 - 5)\ln(Q_\beta + m_e c^2) + \ln\left(\frac{2r_0}{\hbar c}\right)\alpha^2 Z^2 + \frac{1}{3}\ln(A)\alpha^2 Z^2 + \pi\alpha Z$$

Formula 2: $$\ln(T_{1/2}) = a_1 + (\alpha^2 Z^2 - a_2 - a_3 I)\ln(Q_\beta + m_e c^2 - a_4\delta) + \ln\left(\frac{2r_0}{\hbar c}\right)\alpha^2 Z^2 + \frac{1}{3}\ln(A)\alpha^2 Z^2 - a_5\alpha Z$$
$$+ a_6 e^{-[(N-28)^2+(Z-20)^2]/22} + a_7 e^{-[(N-50)^2+(Z-40)^2]/33} + a_8 e^{-[(N-82)^2+(Z-56)^2]/33} + a_9 e^{-[(N-132)^2+(Z-82)^2]/12}$$

	$T_{1/2}<10^6$ s	$T_{1/2}<10^3$ s	$T_{1/2}<1$ s
Formula 1	**1.096**	**0.732**	**0.478**
Formula 2	**0.609**	**0.403**	**0.220**
RHB+QRPA	1.884	1.620	0.463
FRDM+QRPA	0.819	0.597	0.391

J. G. Xia *et al.*, Acta Phys. Sin. 73, 062301 (2024)

Gross theory of β-decay half-lives

- **Gross theory**: based on the **sum rule** of strength function, it treats the β-decay transitions to all final nuclear levels in a **statistical way**

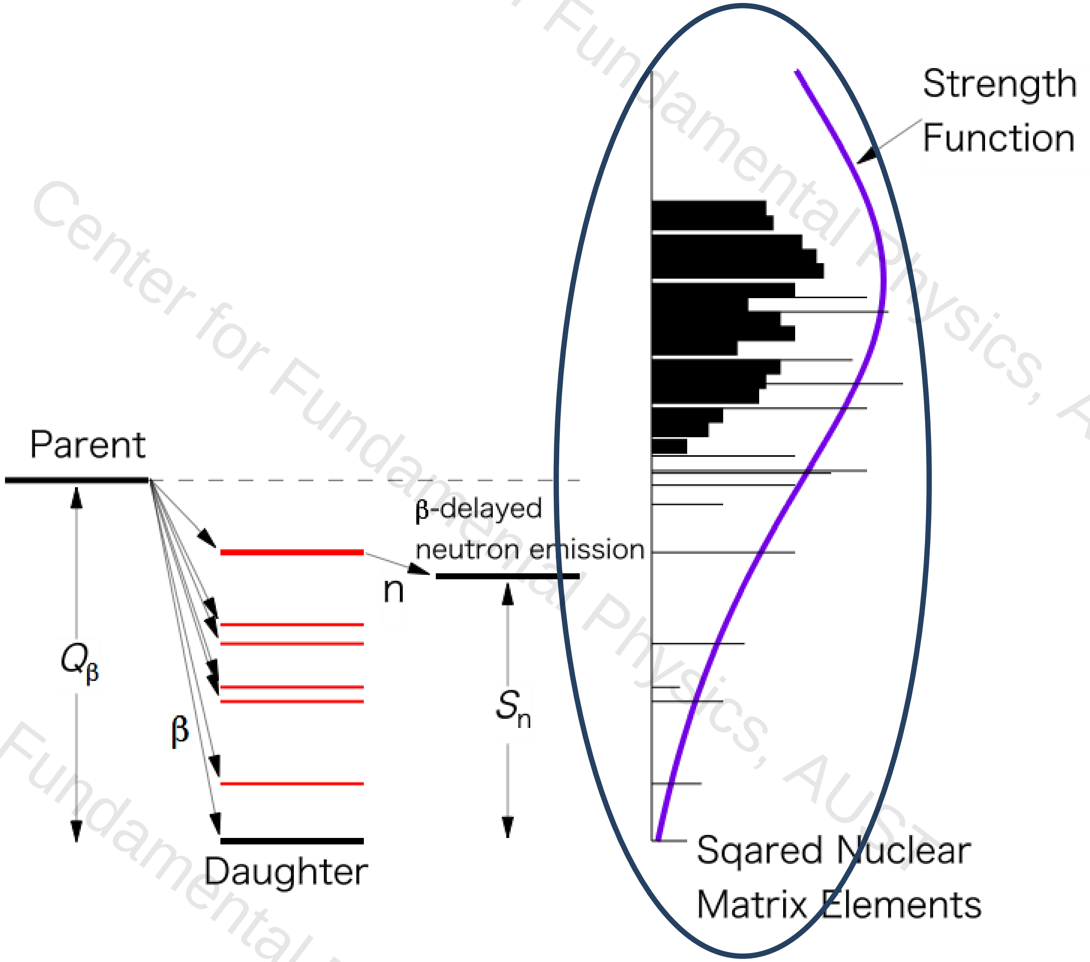
$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{m_e^5 c^4}{2\pi^3 \hbar^7} \sum_E |G_{\mathrm{GT}}|^2 |M_{\mathrm{GT}}(E)|^2 f(-E)$$

$$\approx \frac{m_e^5 c^4}{2\pi^3 \hbar^7} \int_{-Q_\beta}^{0} |G_{\mathrm{GT}}|^2 |M_{\mathrm{GT}}(E)|^2 f(-E)\, dE$$

$$|M_{GT}(E)|^2 = 3\int D_{GT}(E,\varepsilon) \frac{dN_1}{d\varepsilon} W(E,\varepsilon)\, d\varepsilon$$

sum rule

$$\begin{cases} \int_{-\infty}^{\infty} D_{GT}(E,\varepsilon)\, dE = 1 \\ \int_{-\infty}^{\infty} E D_{GT}(E,\varepsilon)\, dE = \Delta E_{GT} \approx \Delta E_C \\ \int_{-\infty}^{\infty} E^2 D_{GT}(E,\varepsilon)\, dE = \sigma_{GT}^2 \approx \sigma_C^2 + \sigma_N^2 \end{cases}$$



Nuclear β-decay half-lives can be calculated only with Q_β from mass models

Gross theory of β-decay half-lives

- Improvement in the GT centroid energy

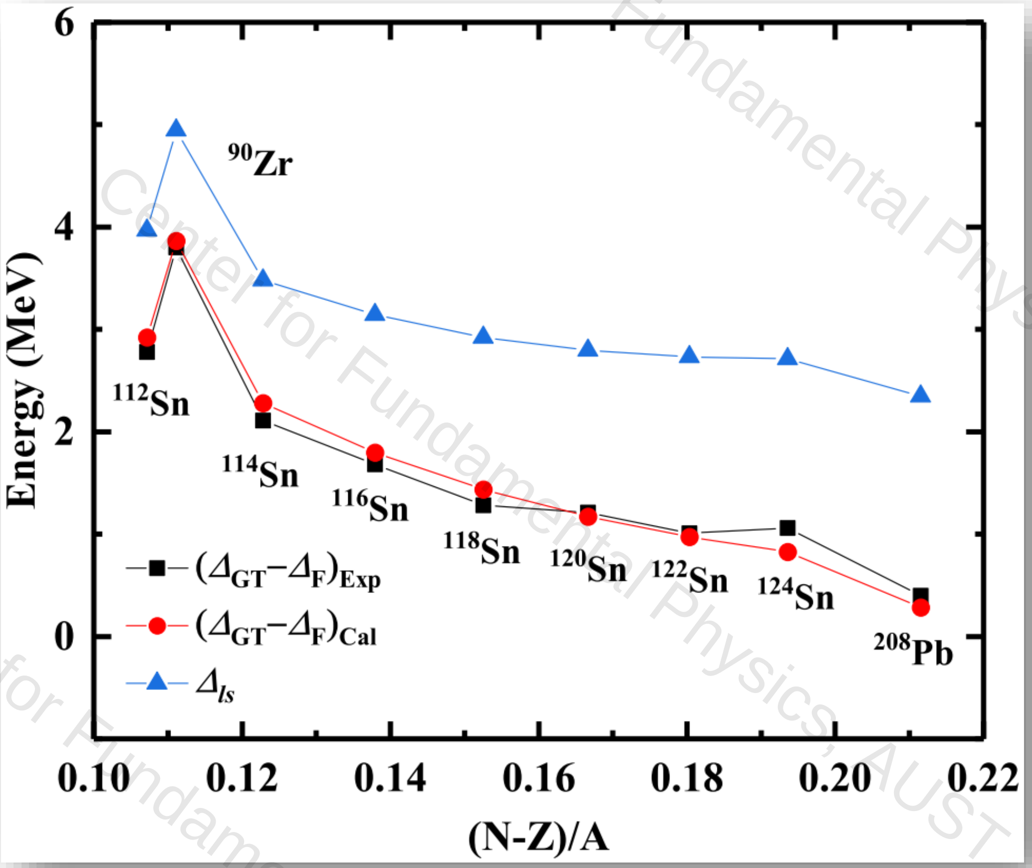
J. Y. Fang *et al.*, Phys. Rev. C 106, 054318 (2022)

$$\frac{\int_{-\infty}^{\infty} E\left|M_\Omega(E)\right|^2 dE}{\int_{-\infty}^{\infty}\left|M_\Omega(E)\right|^2 dE} = \frac{\int_{-\infty}^{\infty} E D_\Omega(E) dE \int_{\varepsilon_{\min}}^{\varepsilon_1} \frac{dN_1}{d\varepsilon} d\varepsilon}{\int_{-\infty}^{\infty} D_\Omega(E) dE \int_{\varepsilon_{\min}}^{\varepsilon_1} \frac{dN_1}{d\varepsilon} d\varepsilon}$$

$$= \int_{-\infty}^{\infty} E D_\Omega(E) dE = \Delta E_{\mathrm{GT}}$$

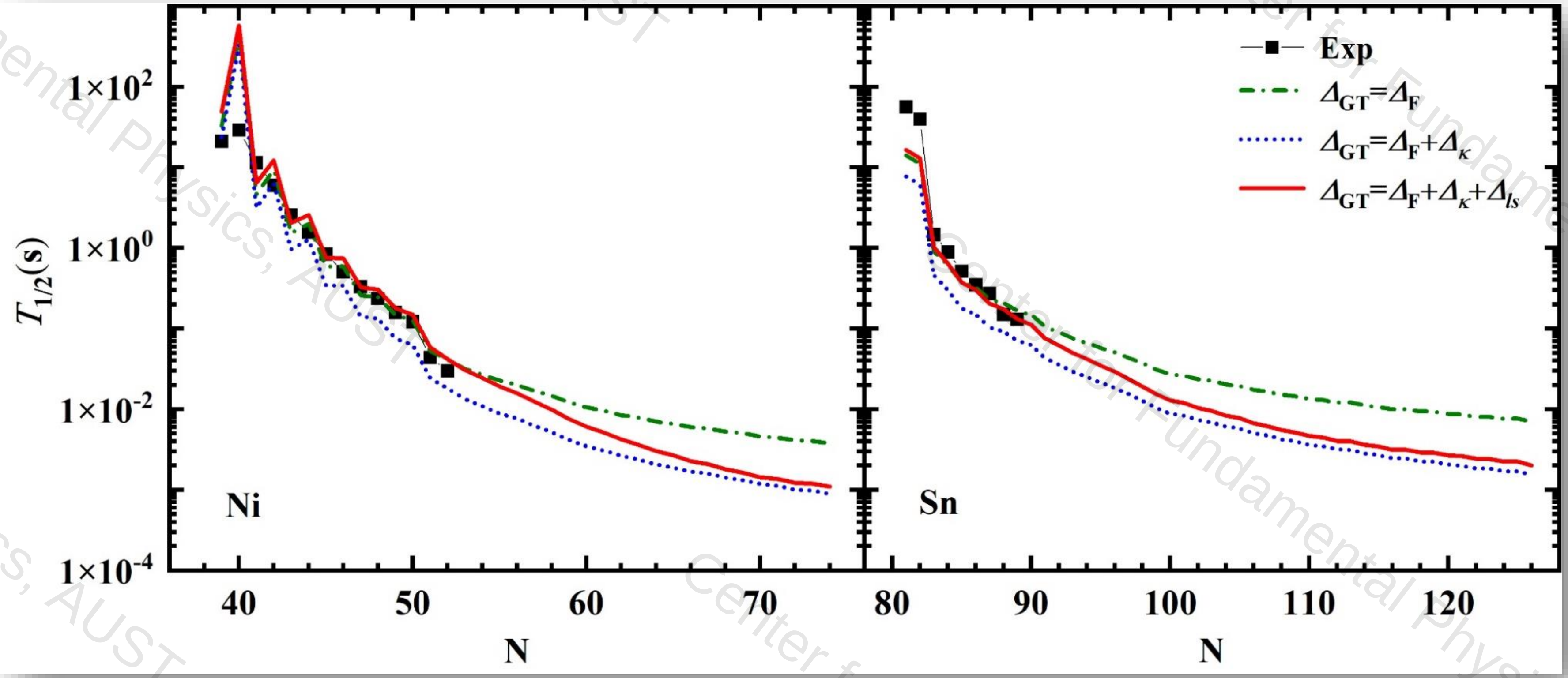
$$\Delta E_{\mathrm{GT}} \approx \Delta E_C \quad \Rightarrow \quad \begin{cases} \varDelta E_{\mathrm{GT}} = \varDelta E_C + \varDelta_\kappa + \varDelta_{ls} \\ \varDelta_\kappa = 2\left(\kappa_{\sigma\tau} - \kappa_\tau\right)\frac{N-Z}{A} \\ \varDelta_{ls} = \frac{2}{3(N-Z)} E_{ls} \end{cases}$$

$$E_{ls} = \sum_i \frac{\varDelta E_i\left[\left(u_{p^-}^2 v_{n^+}^2\right)\mu_{p^-}\mu_{n^+} - \left(u_{p^+}^2 v_{n^-}^2\right)\mu_{p^+}\mu_{n^-}\right]_i}{2l_i + 1}$$



Spin-orbit splitting extracted by CDFT is essential for the reliable description of GT centroid energy!

β-decay half-lives from the gross theory

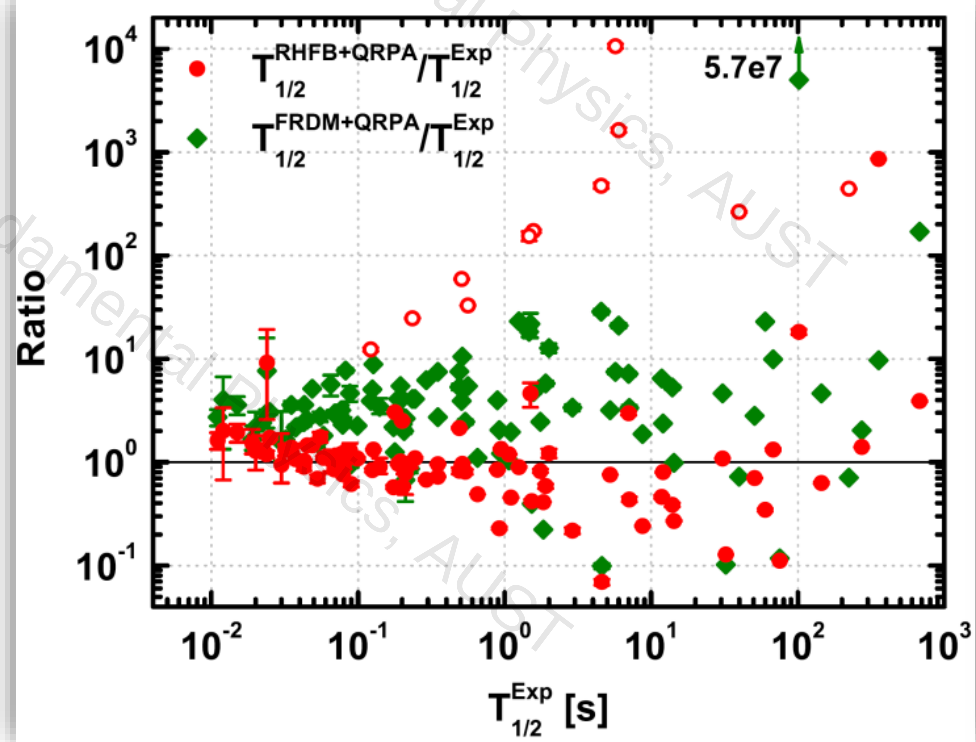


	WS4+GT	SHFB+FAM	RHB+QRPA	FRDM12+QRPA
$\sigma_{rms}(\log_{10}T_{1/2})$	**0.40**	0.62	0.80	0.53

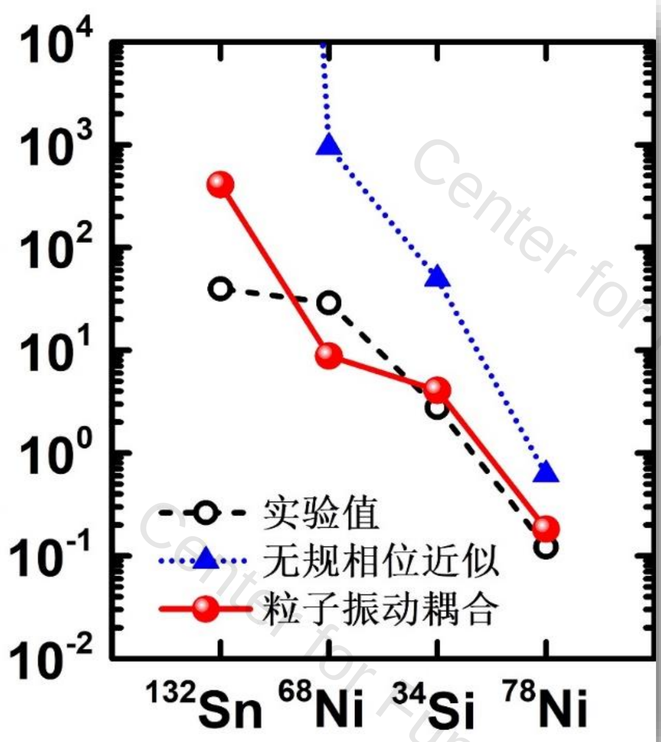
The improved gross theory is more accurate than other models and significantly reduces the β-decay half-lives of neutron-rich nuclei

β-decay half-lives from the RQRPA approach

β^--decay half-lives

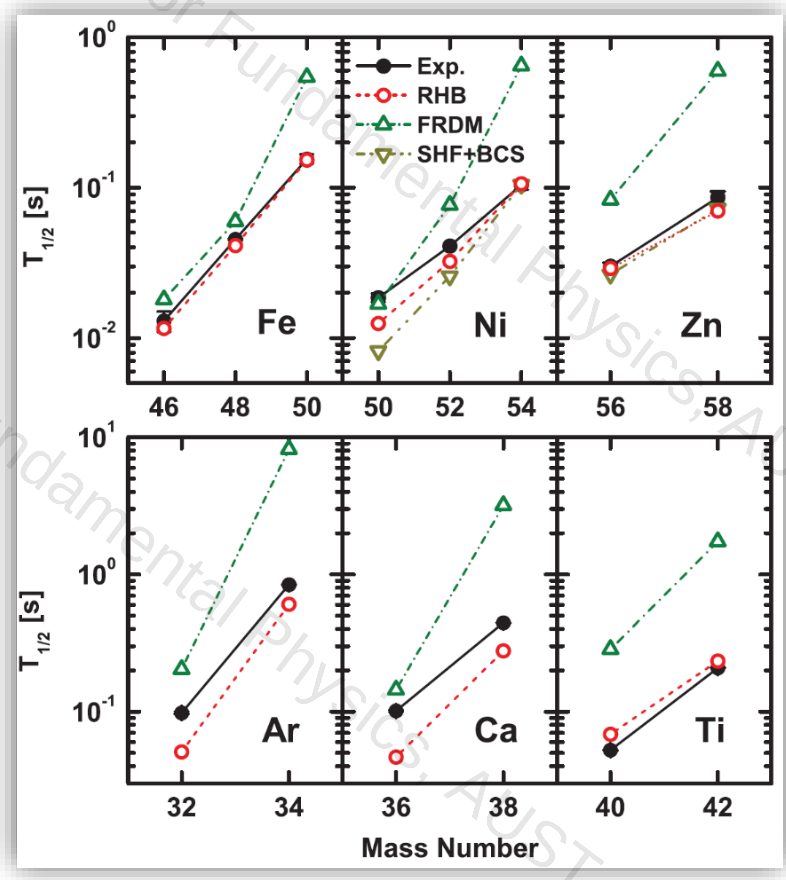


Z.M. Niu *et al.*, PLB 723, 172 (2013)



Y.F. Niu *et al.*, PRL 114, 142501 (2015)

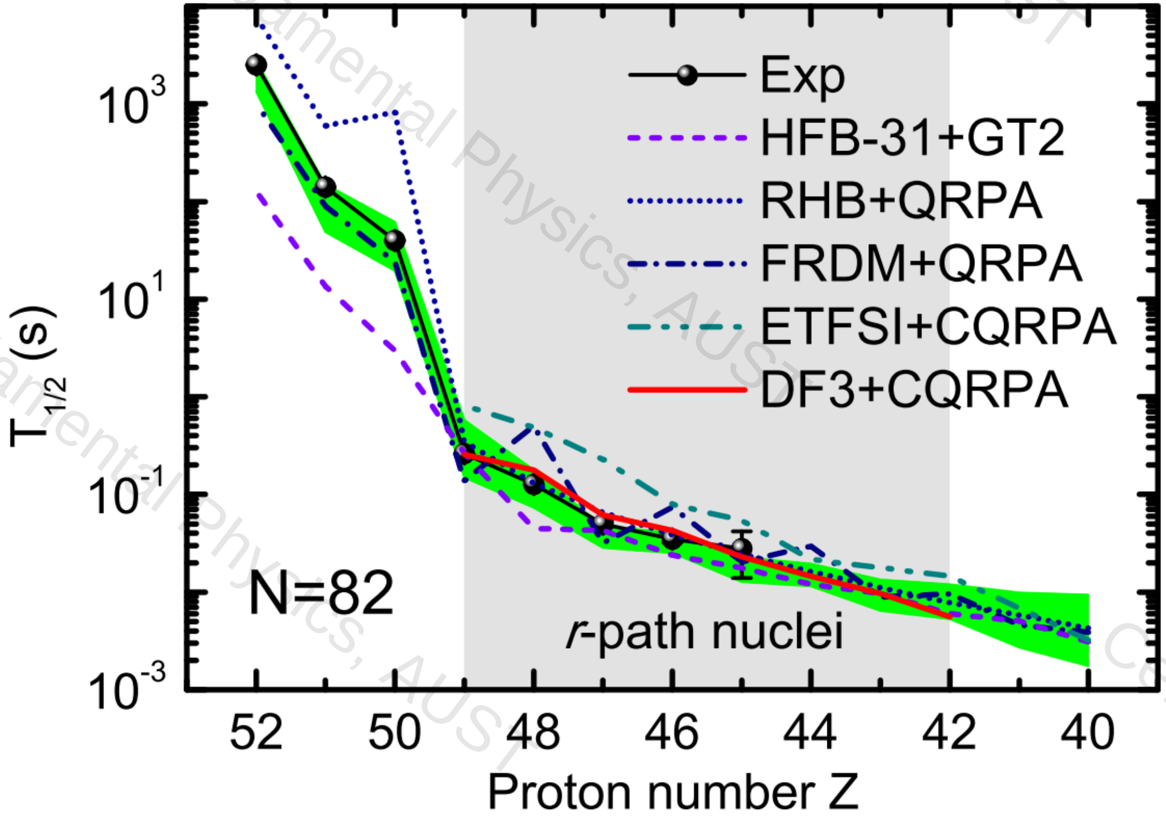
β^+/EC-decay half-lives



Z.M. Niu *et al.*, PRC 87, 051303(R) (2013)

- The self-consistent RQRPA approach was developed and it well reproduces the experimental β-decay half-lives for both even-even neutron-rich and neutron-deficient nuclei.

β-decay half-lives from machine learning

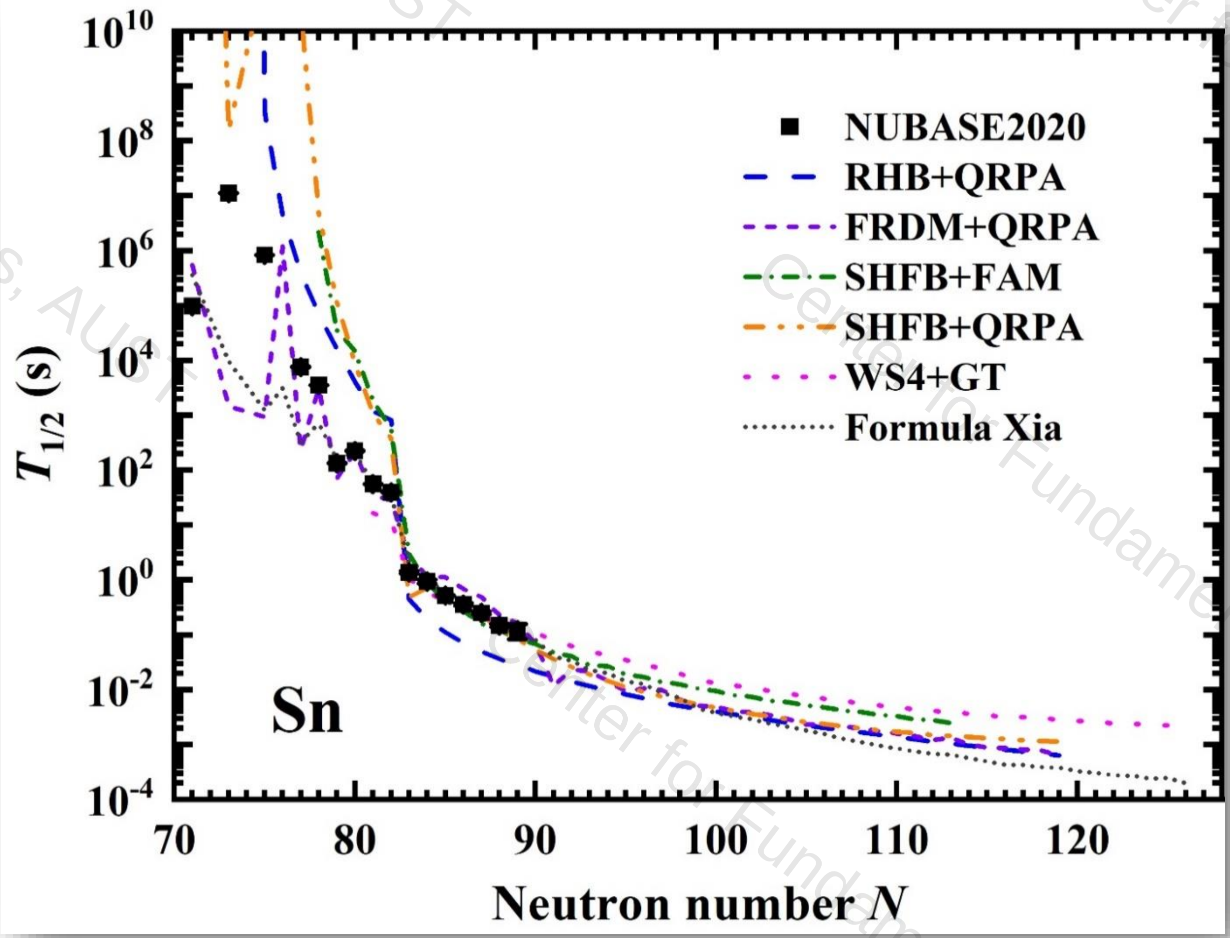


	$T_{1/2}<10^6$ s	$T_{1/2}<10^3$ s	$T_{1/2}<1$ s
a/f	0.806	0.630	0.563
BNN-I2	0.477	0.354	0.238
BNN-I4	**0.400**	**0.315**	**0.204**
FRDM+QRPA	0.819	0.597	0.391
RHB+QRPA	1.884	1.620	0.463

- The BNN-I4 approach well reproduces the experimental data, even completely agree with the experimental data within uncertainties for short-lived nuclei.
- When extrapolate from known region, the results of other models generally agree with BNN-I4 predictions within uncertainties.

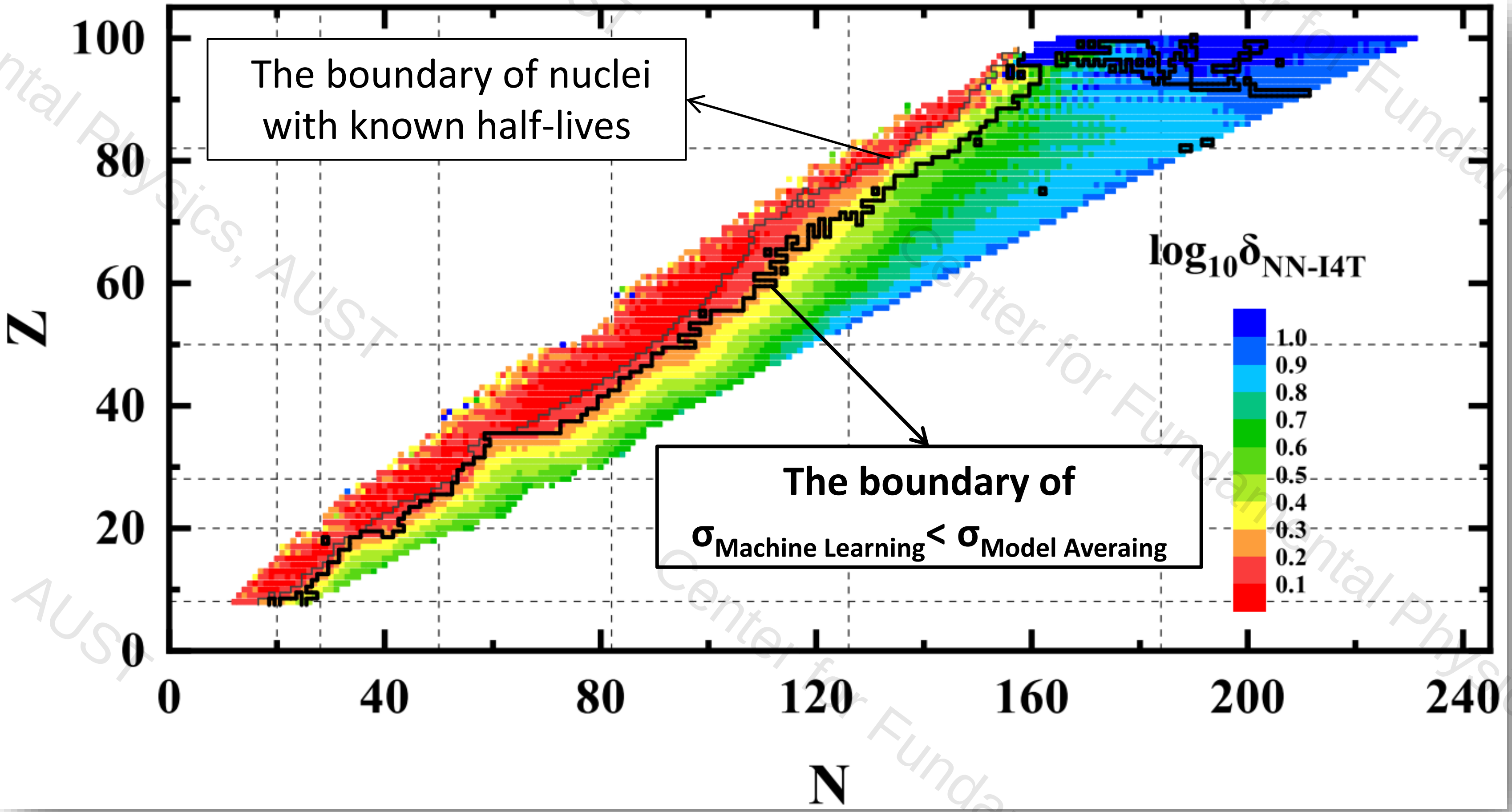
Z. M. Niu *et al.*, PRC 99, 064307 (2019)

Nuclear models for β-decay half-lives



- Different models generally better reproduce β-decay half-lives of short-lived nuclei. The deviations between different predictions slowly increase to an order of magnitude even out to the drip line.

β-decay half-lives from machine learning



- The half-life uncertainties of the neural network are still smaller than those of the model averaging method within about 5–10 steps for nuclei with $35 \lesssim Z \lesssim 90$. W. F. Li *et al.*, J. Phys. G 51, 015103 (2024)

Outline

Effect of mass uncertainty on r-process abundances

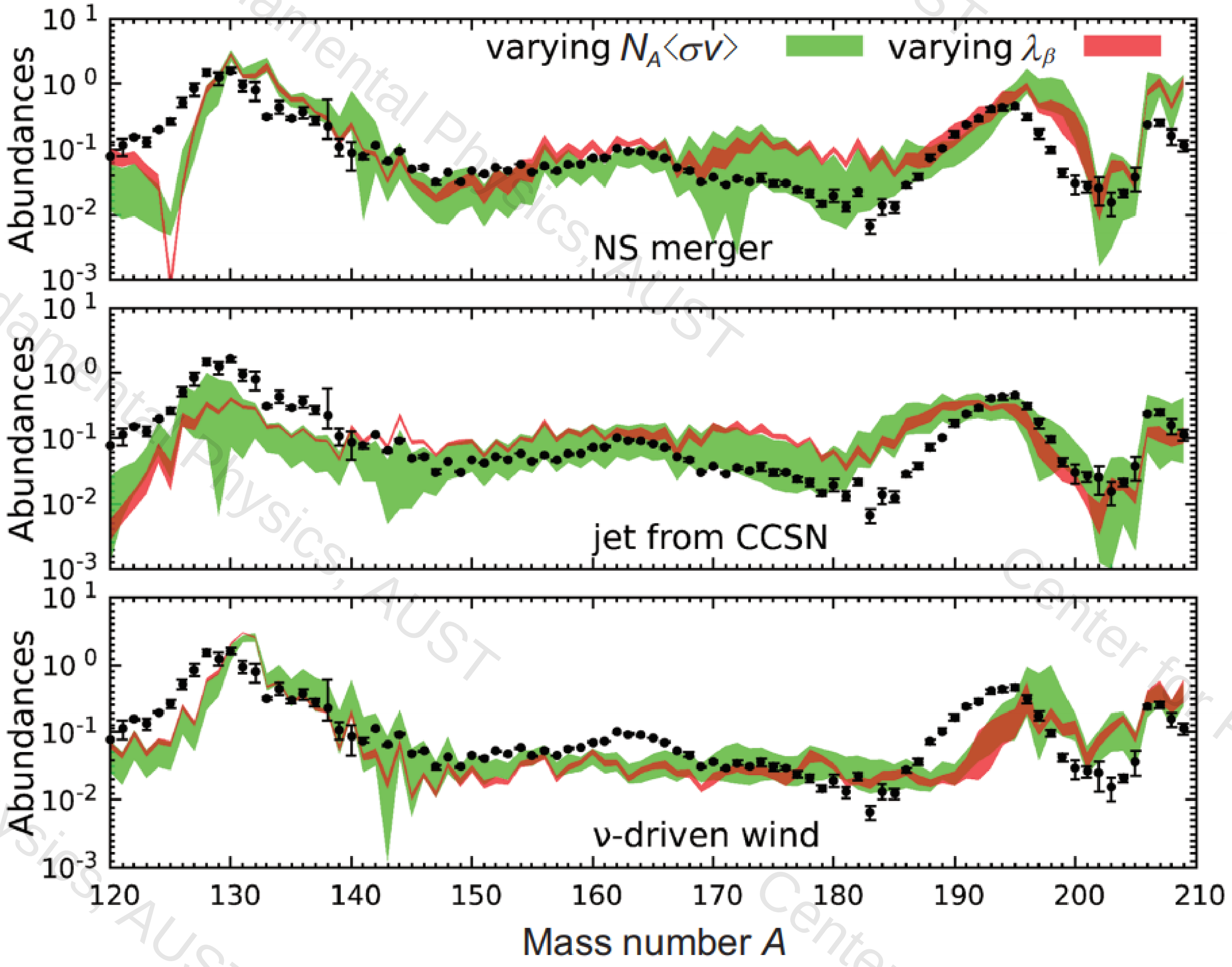


Fig: Green and red bands show the uncertainties in the systematic simulations with various neutron-capture rates and β-decay rates, respectively.

- **Mass predictions** from 10 mass models:
  - ★ Empirical formula: **β-decay half-lives**
  - ★ TALYS code: **neutron-capture rates**

- The final r-process abundances include uncertainties introduced by the nuclear mass model **mainly through the variation of neutron-capture rates**, whereas the uncertaintiesof β-decay rates make a relatively small contribution.

Z. Li *et al.*, SCPMA 62, 982011 (2019)

Effect of half-life uncertainty on freeze-out time

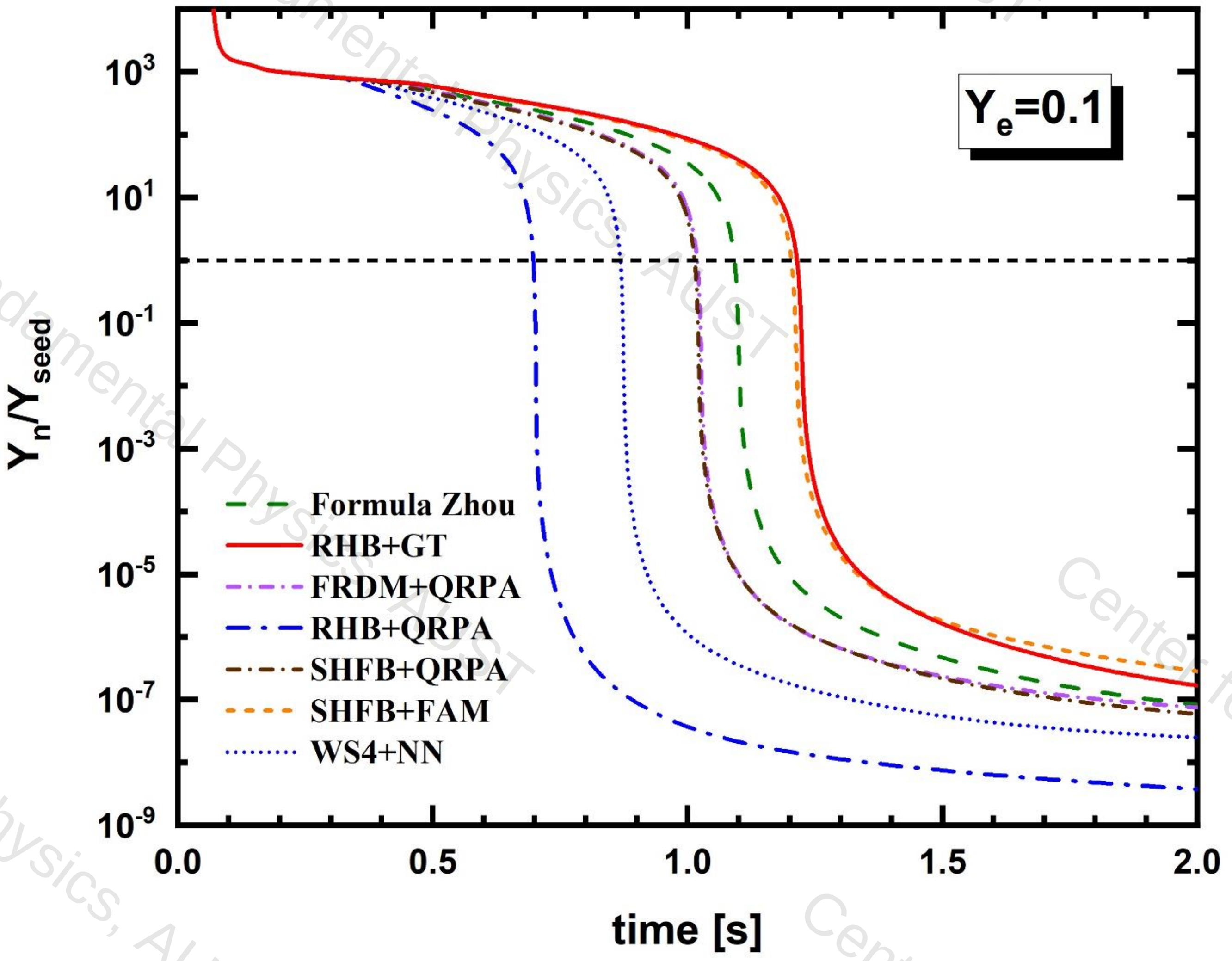


Fig: Evolution of Y_n/Y_{seed} as a function of time in the *r*-process simulations with different β-decay half-life predictions.

- Y_n/Y_{seed} decreases very quickly around the freeze-out time.
- **Nuclear β-decay half-lives have a significant effect on the freeze-out time.**
- The freeze-out time is between 0.7-1.2 s.

Outline

1. Introduction
2. Nuclear physics inputs
   - ⋆ Nuclear masses
   - ⋆ Nuclear β-decay half-lives
3. r-process simulations
4. Summary and perspectives

Summary and perspectives

- Summary:
  - ⋆ Various theoretical models and machining learning methods are developed to predict nuclear masses and β-decay half-lives, and the accuracies are remarkably improved.
  - ⋆ The uncertainties of r-process abundances introduced by the nuclear mass uncertainties mainly through the variation of neutron-capture rates, while β-decay half-lives play an important role in determine the time scale of r-process.
- Perspectives:
  - ⋆ Develop theoretical models and machining learning methods to improve the accuracies of nuclear mass and β-decay half-life by including more and more physics effects.
  - ⋆ Constrcut high-precision nuclear inputs for r-process simulations including nuclear masses, half-lives, and neutron-capture rates.

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合作者：

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西南大学：李志攀

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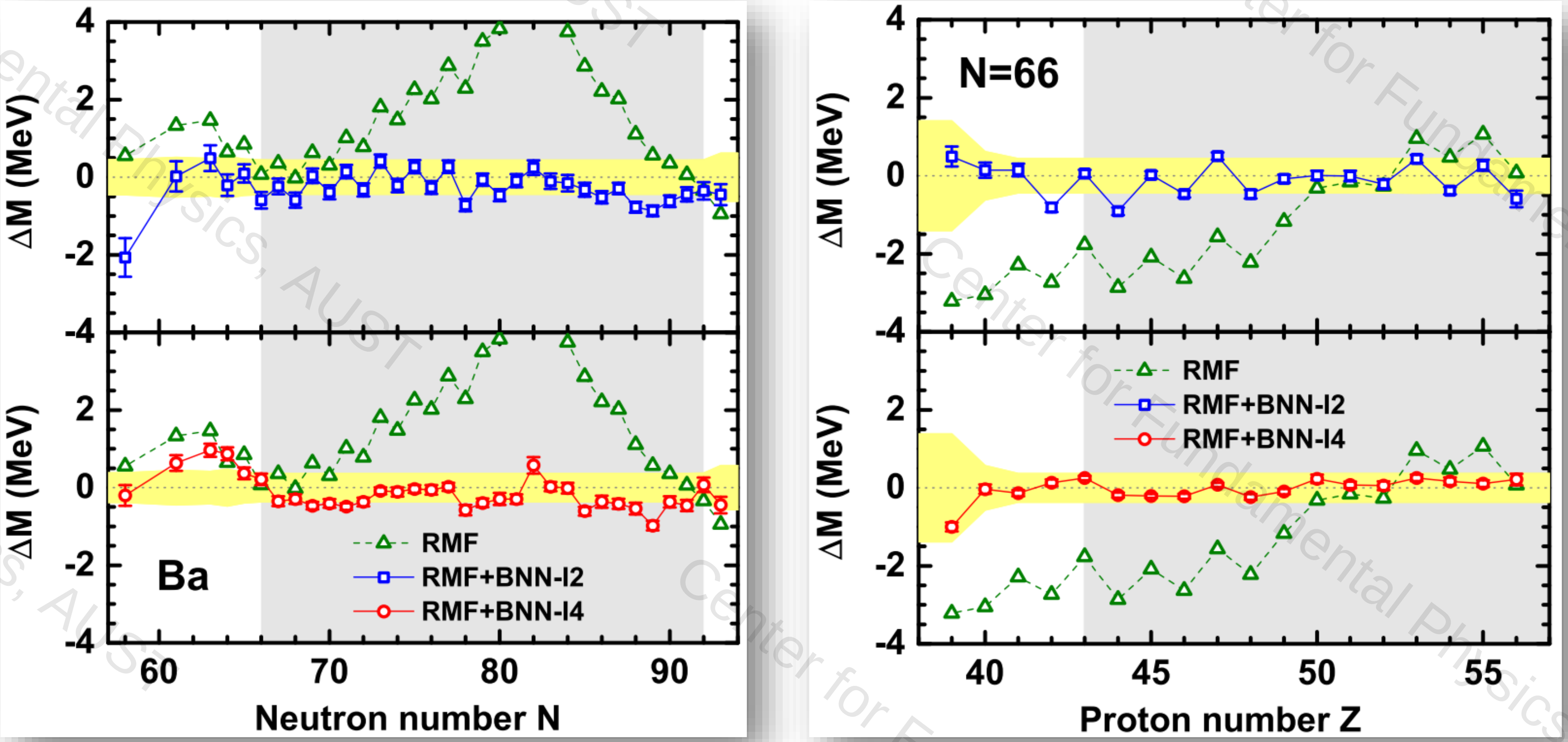
Thank you!

Mass extrapolation



- The smooth deviations can be improved with both BNN approaches, while **the odd-even staggering can only remarkably reduced with BNN-I4 approach**.
- The BNN corrections are still reasonable if the extrapolation is not far away from the training region. Z.M. Niu and H.Z. Liang, PLB 778, 48 (2018)

Mass predictions of RMF+BNN model

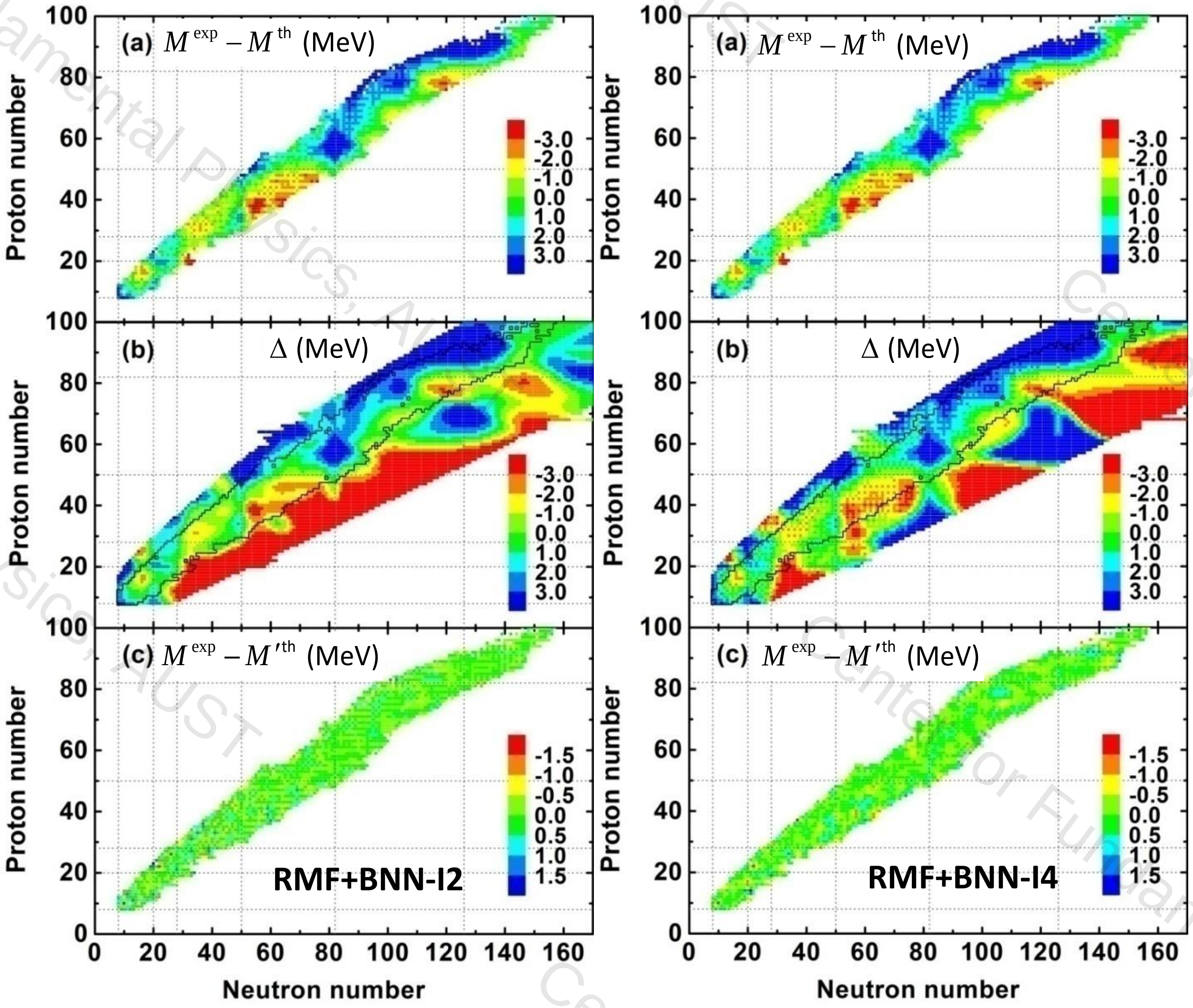
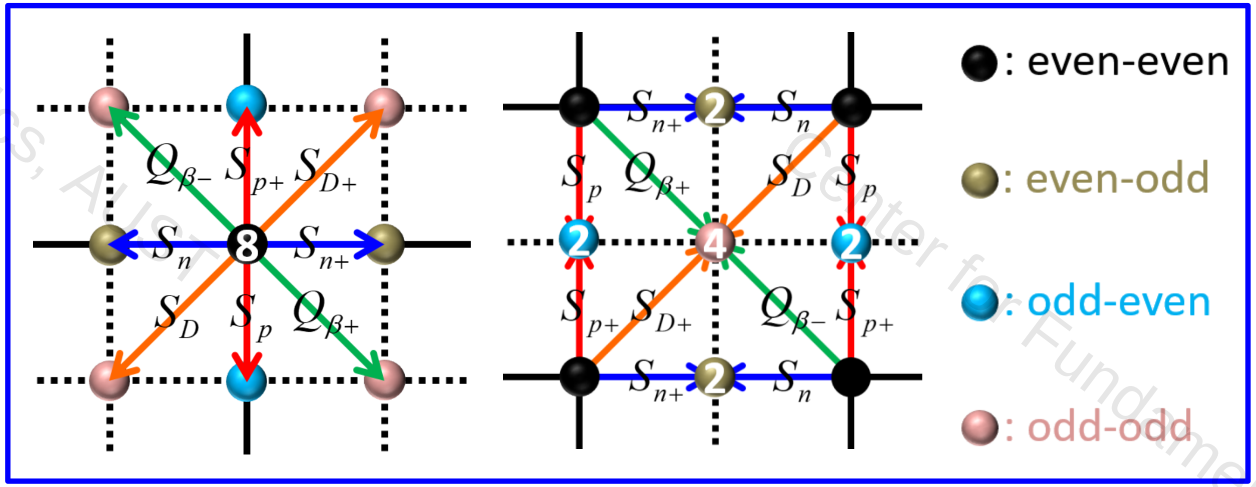


Figure: (a) Mass differences between the experimental data in AME16 and the predictions of the RMF model. (b) BNN corrections. (c) Mass differences after BNN improvement. Niu and Liang, PLB 778, 48 (2018)

- Smooth mass deviations can be easily removed by both BNN approaches, while the odd-even staggering can be well reproduced only using BNN-I4 approach.
- The extrapolation of BNN correction show more structure information for the BNN-I4 approach, especially the shell effects around (Z,N)=(28, 82) and (50, 126).

Idea of BML

- Input $\delta = [(-1)^Z + (-1)^N]/2$



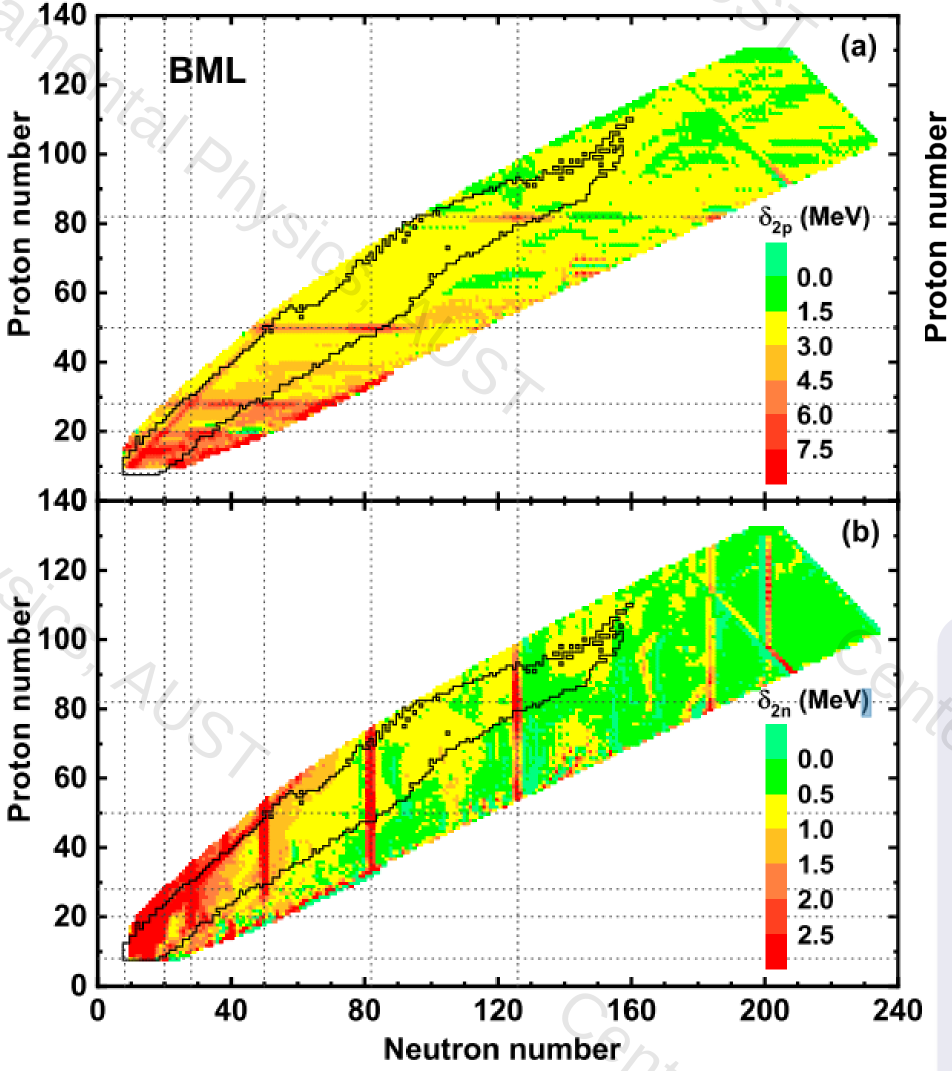
$$\bar{M}(Z,N) = M(Z,N);\ \bar{M}(Z+1,N+1) = \sum_{i=1}^{4} M^i(Z+1,N+1)/4$$

$$\bar{M}(Z,N+1) = \sum_{i=1}^{2} M^i(Z,N+1)/2;\ \bar{M}(Z+1,N) = \sum_{i=1}^{2} M^i(Z+1,N)/2$$

- Input $P = \nu_n \nu_p / (\nu_p + \nu_n),\ \nu_p = \min(|Z - Z_0|),\ \nu_n = \min(|N - N_0|)$

$$E_{\rm mic}^{\rm model} = M^{\rm model} - E_{\rm mac}^{\rm FRDM12} \text{ or } E_{\rm mic}^{\rm model} = M^{\rm model} - E_{\rm mac}^{\rm LDM}$$

BML predictions



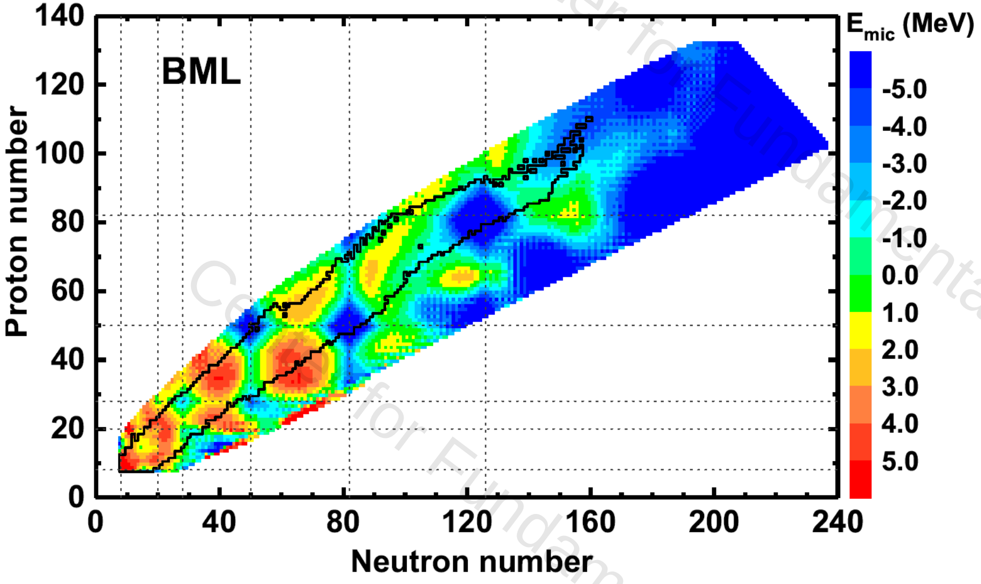


Fig: δ_{2p}, δ_{2n}, and E_{mic} of BML.

Z.M. Niu and H.Z. Liang, PRC 106, L021303 (2022)

★ The shell structure in the known region is well reproduced.

★ Several important features in the unknown region are predicted, such as the magic numbers around **N=40 and N=184**, the **robustness of N=82 shell**, the **quenching of N = 126 shell**.

BML predictions

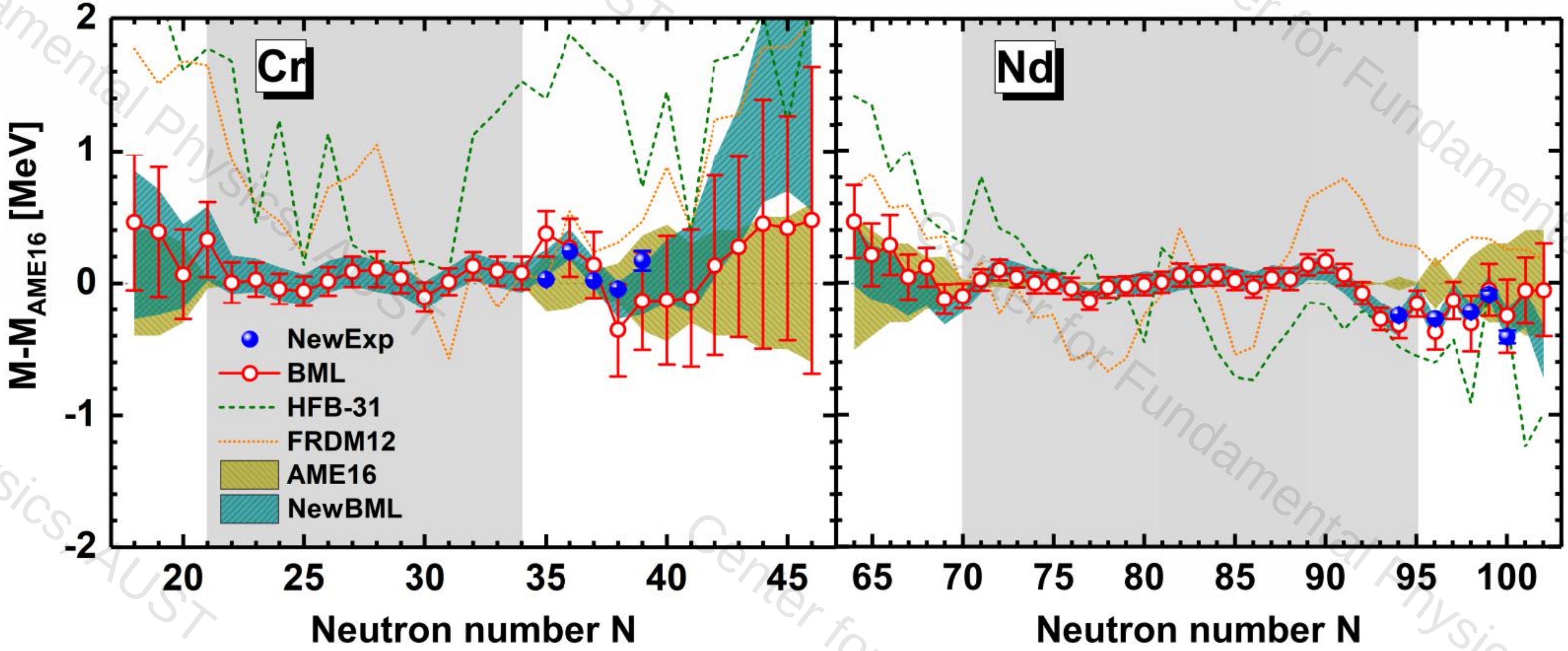


Fig: Mass differences between M_{exp} in AME16 and M_{th} from BML, NewBML, HFB-31, and FRDM12 models. The new experimental data from [Mougeot2018PRL; Orford2018PRL] are denoted by spheres.

★ The BML model well reproduces new experimental masses within errors. If new data are included in Lset, the errors near new data reduce to about half of original values.

Gross theory of β-decay half-lives

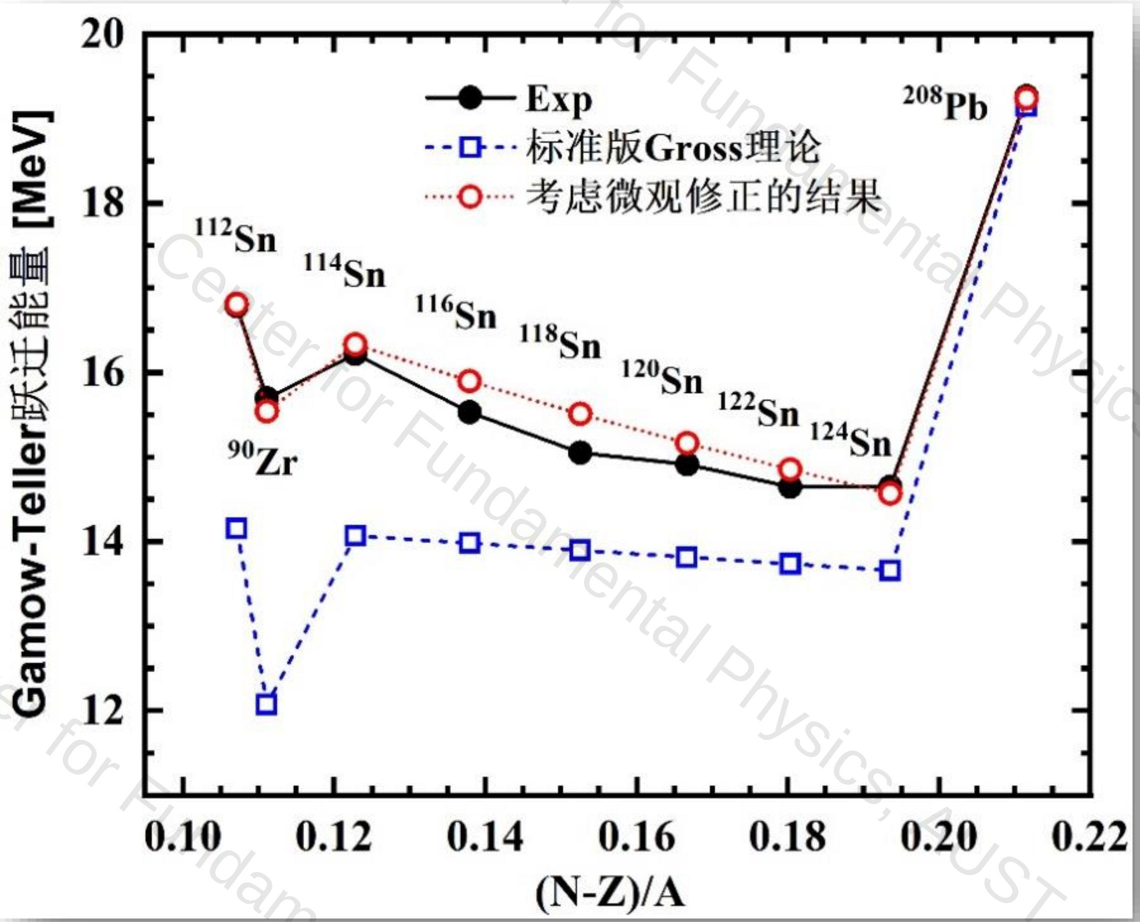
- Improvement in the GT centroid energy

$$\frac{\int_{-\infty}^{\infty} E\left|M_\Omega(E)\right|^2 dE}{\int_{-\infty}^{\infty}\left|M_\Omega(E)\right|^2 dE}=\frac{\int_{-\infty}^{\infty} ED_\Omega(E)dE\int_{\varepsilon_{\min}}^{\varepsilon_1}\frac{dN_1}{d\varepsilon}d\varepsilon}{\int_{-\infty}^{\infty} D_\Omega(E)dE\int_{\varepsilon_{\min}}^{\varepsilon_1}\frac{dN_1}{d\varepsilon}d\varepsilon}$$

$$=\int_{-\infty}^{\infty} ED_\Omega(E)dE=\Delta E_{GT}$$

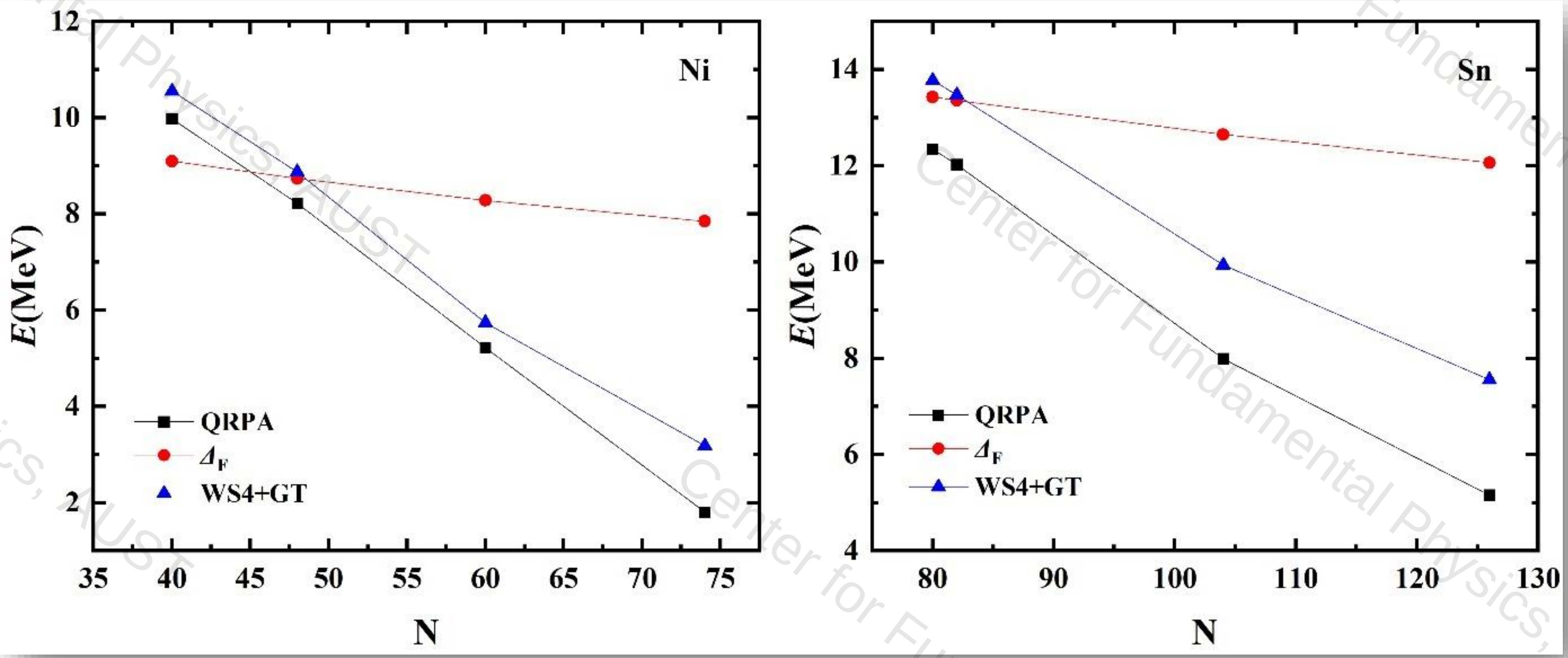
$$\Delta E_{GT}\approx\Delta E_C\quad\Rightarrow\quad\begin{cases}\Delta E_{GT}=\Delta E_C+\Delta_\kappa+\Delta_{ls}\\ \Delta_\kappa=2\left(\kappa_{\sigma\tau}-\kappa_\tau\right)\dfrac{N-Z}{A}\\ \Delta_{ls}=\dfrac{2}{3(N-Z)}E_{ls}\end{cases}$$

$$E_{ls}=\sum_i\frac{\Delta E_i\left[\left(u_{p^-}^2v_{n^+}^2\right)\mu_{p^-}\mu_{n^+}-\left(u_{p^+}^2v_{n^-}^2\right)\mu_{p^+}\mu_{n^-}\right]_i}{2l_i+1}$$



协变密度泛函理论提取的自旋—轨道劈裂对可靠描述GT跃迁能量至关重要！

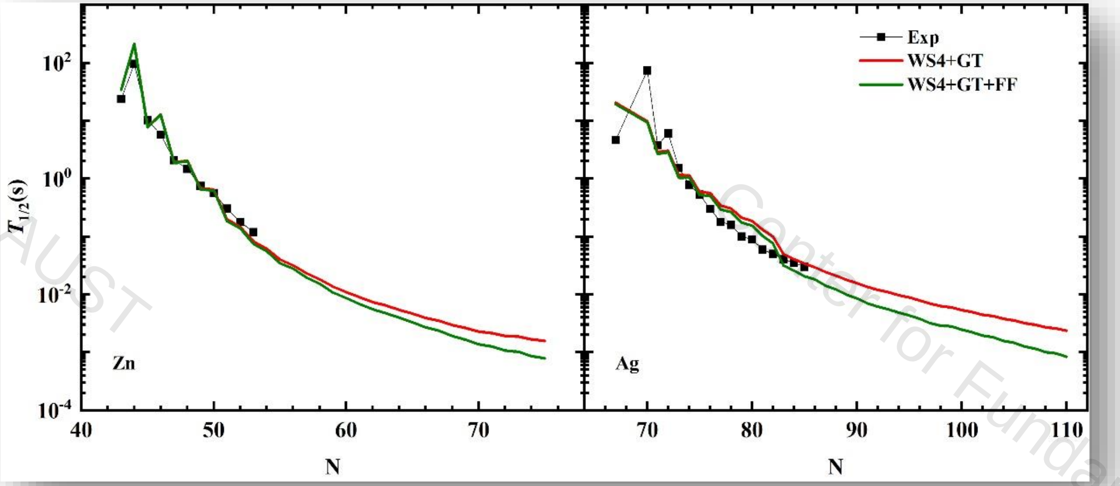
Gross theory

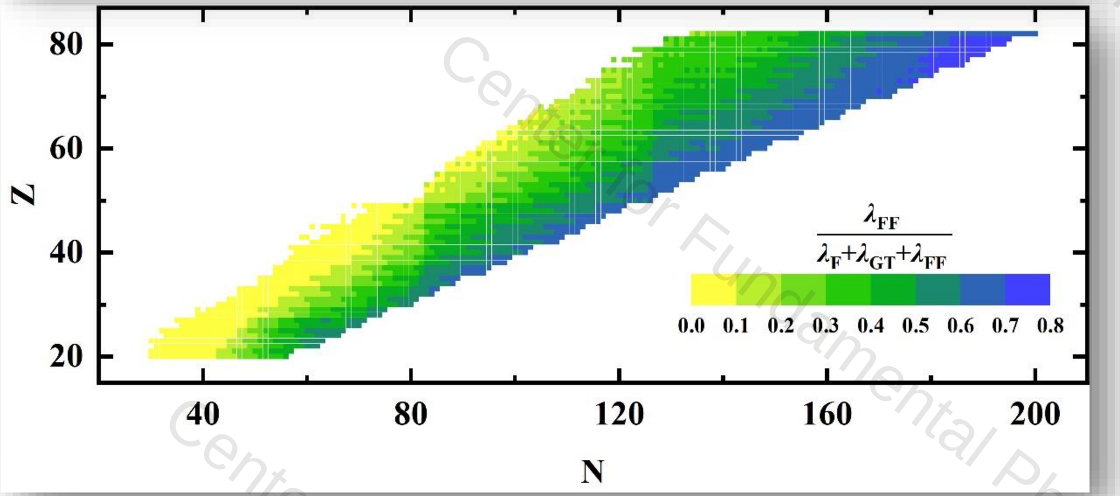


在近滴线核区，新提出的 Gamow-Teller 跃迁中心能量与微观 QRPA 的结果更为接近。

Gross theory

- 一级禁戒跃迁的贡献：





一级禁戒跃迁对重核区丰中子原子核的β衰变寿命贡献较大

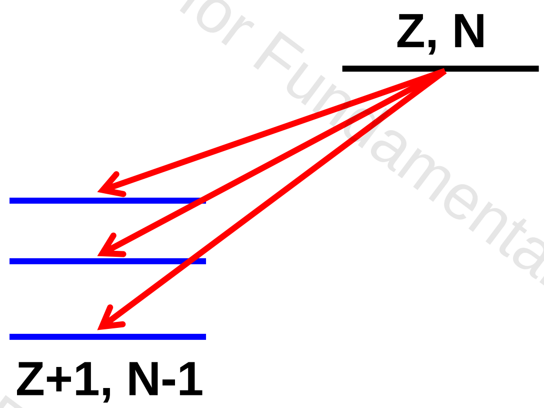
Nuclear β-decay half-lives

- The nuclear β-decay half-life in allowed GT approximation reads as follows:

$$T_{1/2}=\frac{\ln 2}{\lambda_\beta}=\frac{D}{g_A^2\sum_m B_{\rm GT}(E_m)f(Z,A,E_m)}$$

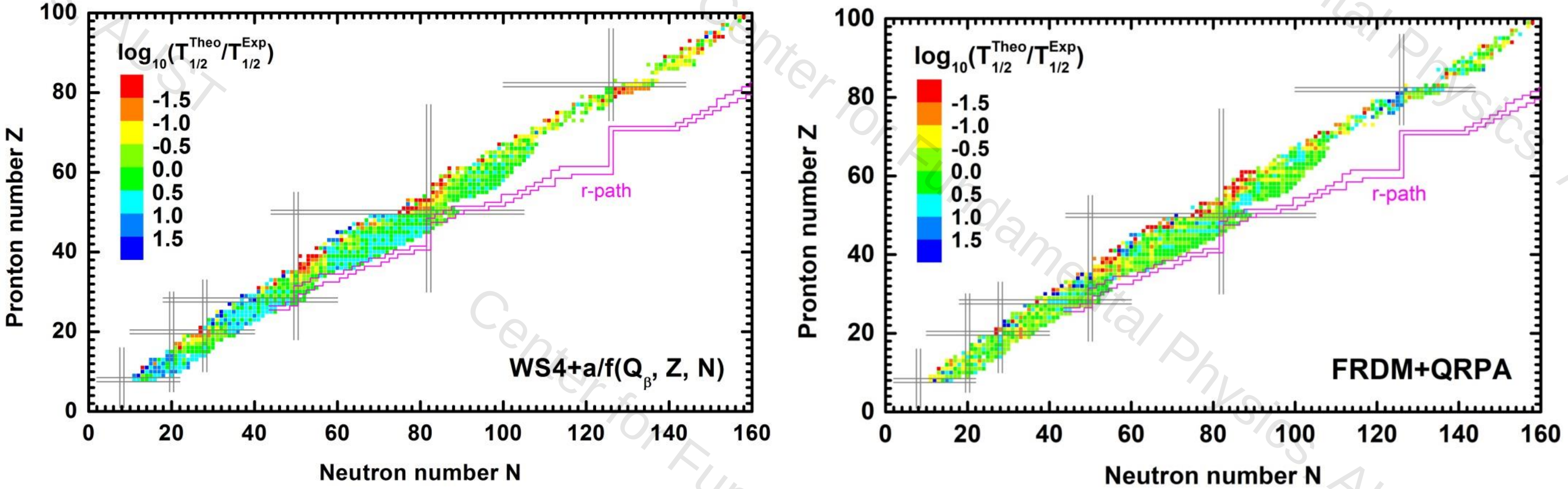
$$\rightarrow T_{1/2}=a/f(Z,A,E_m=Q_\beta-c(\delta-1)/\sqrt{A})$$

where $D=\frac{\hbar^7 2\pi^3\ln 2}{g^2 m_e^5 c^4}=6163.4$ s, $g_A=1$, $B_{\rm GT}(E_m)$ is the transition probability, and E_m is the maximum value of β-decay energy.



The phase volume is

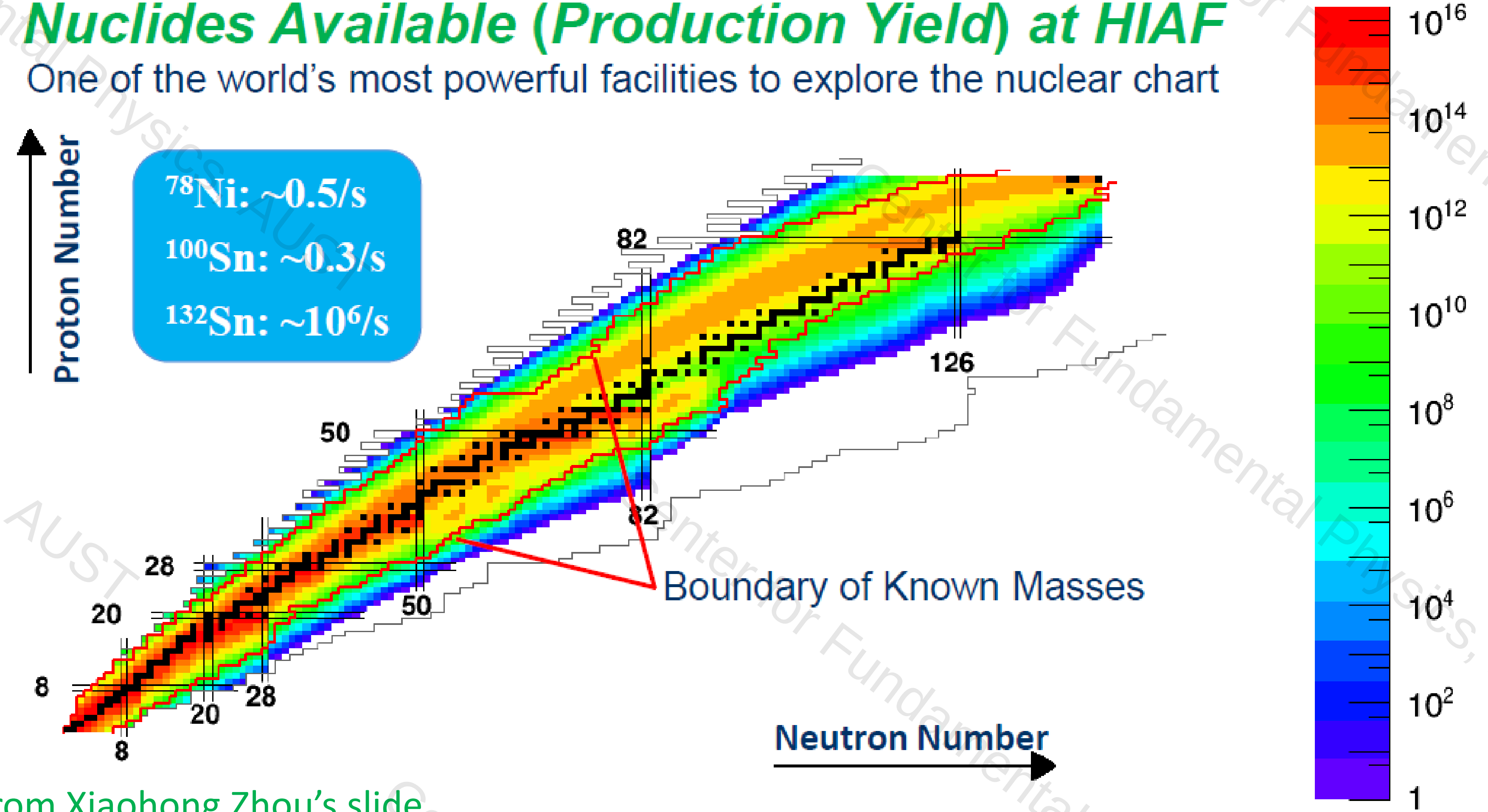
$$f(Z,A,E_m)=\frac{1}{m_e^5}\int_{m_e}^{E_m} p_e E_e (E_m-E_e)^2 F(Z,A,E_m)dE_e,$$



Capability of Producing Nuclides

Nuclides Available (Production Yield) at HIAF

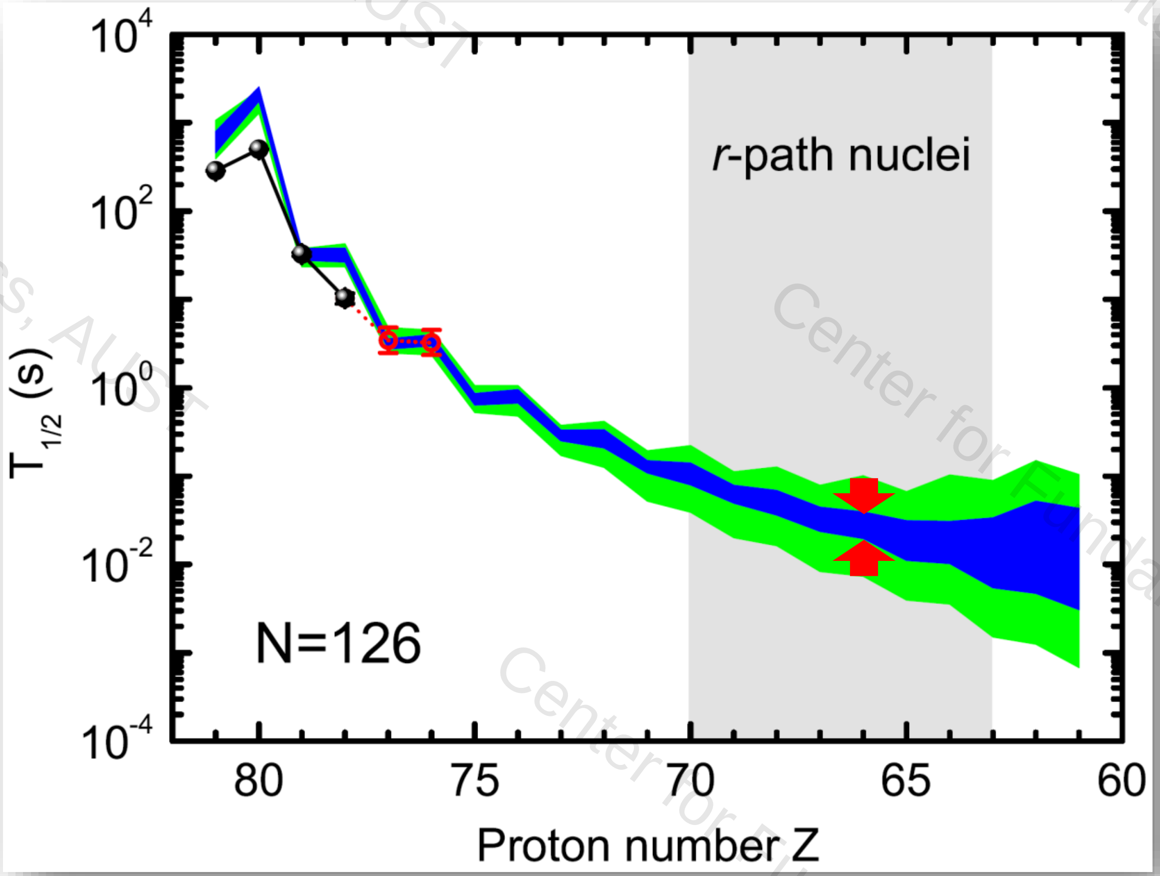
One of the world's most powerful facilities to explore the nuclear chart



From Xiaohong Zhou's slide

Prolific sources of nuclides far away from the stability line will be provided using projectile fragmentation, in-flight fission, multi-nucleon transfer, and fusion reactions. The limits shown are the production rate of one nuclide per day, which enable the "discovery experiments"

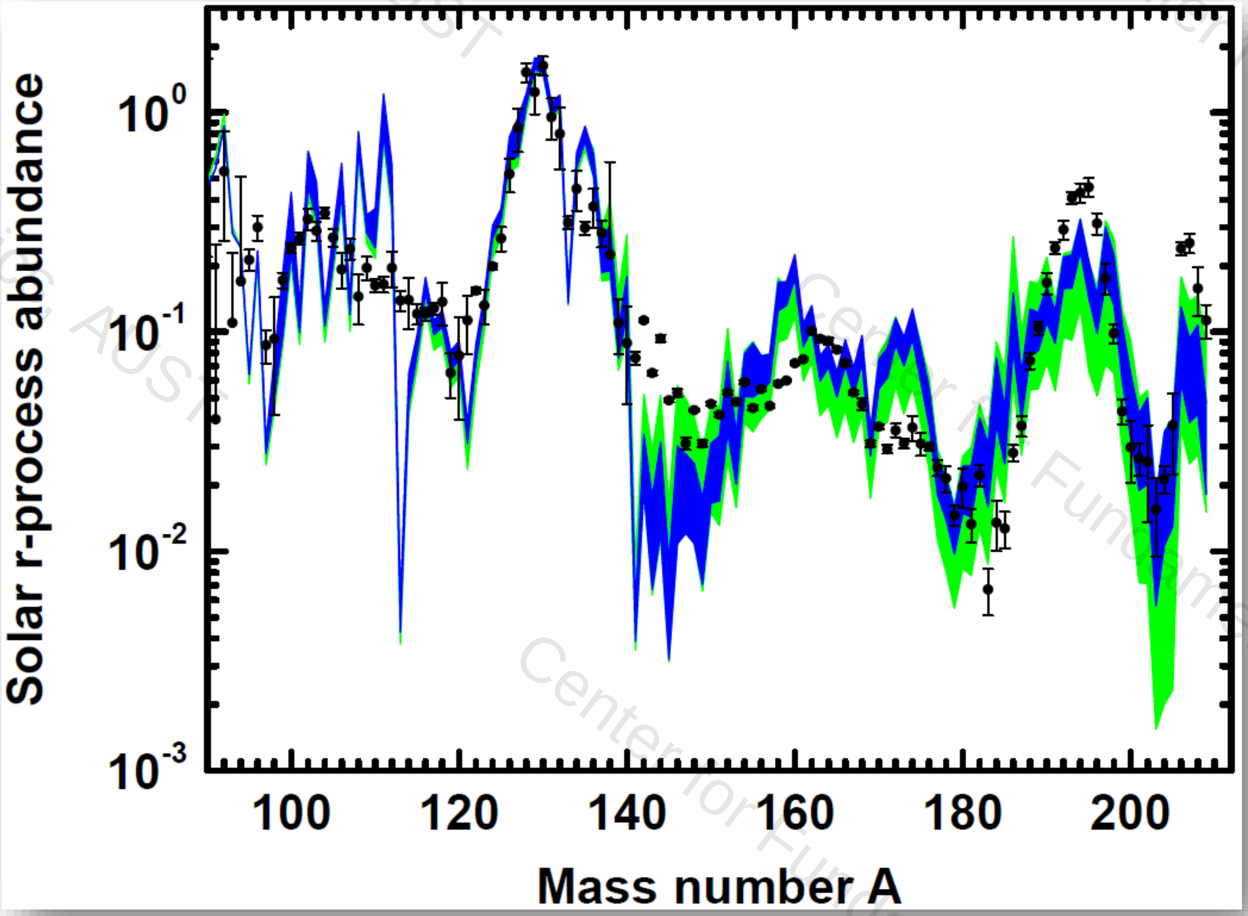
Predictions of nuclear half-lives



- If we can further measure three more β-decay half-lives for each isotopes
  - ✓ uncertainties of BNN predictions are similar in the training region
  - ✓ they will be decreased about 3 times when extrapolate to the region far from known region.

Z. M. Niu et al., PRC 99, 064307 (2019)

Predictions of r-process abundances



- Uncertainties from β-decay half-lives lead to large uncertainties for the *r*-process abundances of elements with A>~140, which can be remarkably reduced if we can further measure three more β-decay half-lives. Z. M. Niu et al., PRC 99, 064307 (2019)

Half-life predictions of NN

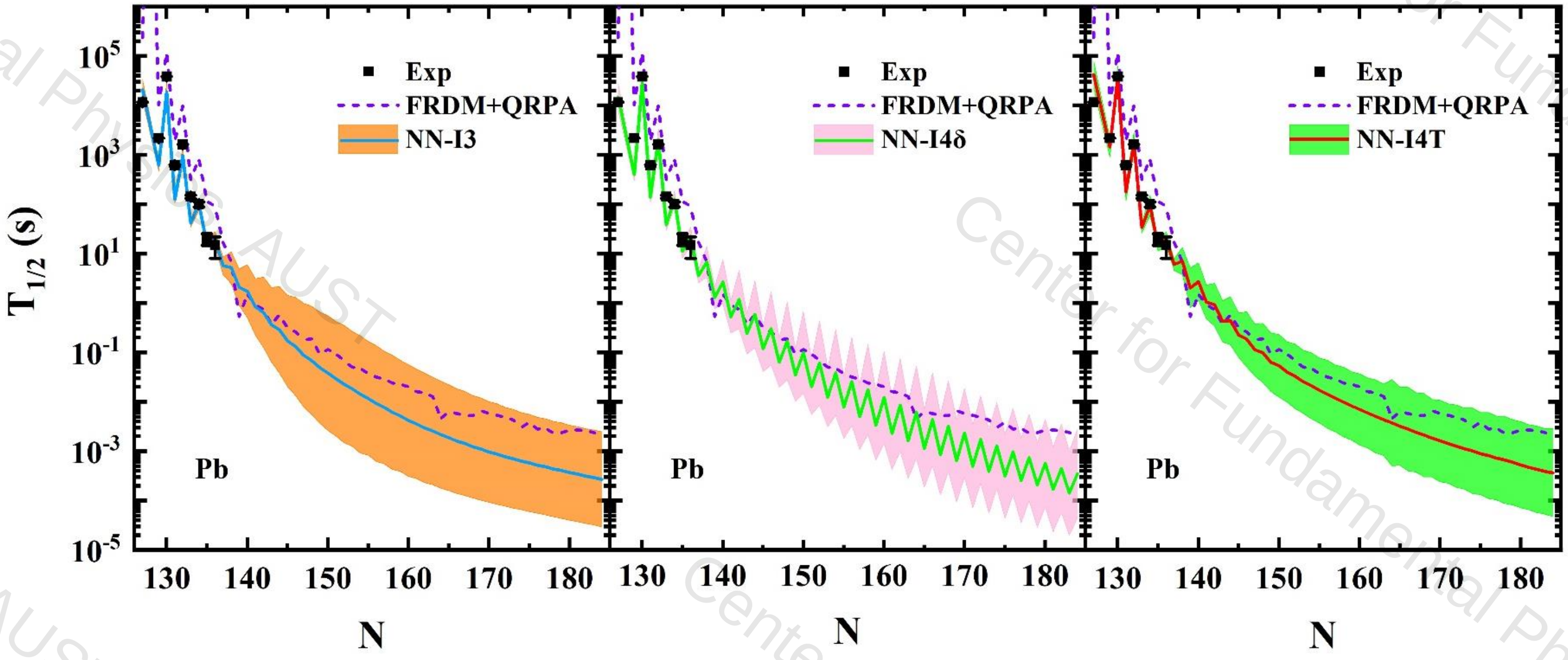


Fig. 2 Nuclear *β*-decay half-lives of Pb isotopes. The experimental half-lives in NUBASE2020 are denoted by squares. For comparison, the theoretical results from FRDM + QRPA are shown by the dashed line.

- ◆NN-I4δ: There are obvious odd-even staggering in the region far away from the known nuclei.
- ◆The odd - even staggering predicted can be eliminated by removing δ input from the neural network input layer.
- ◆ uncertainties : NN-I4T < NN-I3 in the unknown region.

Half-life predictions of NN

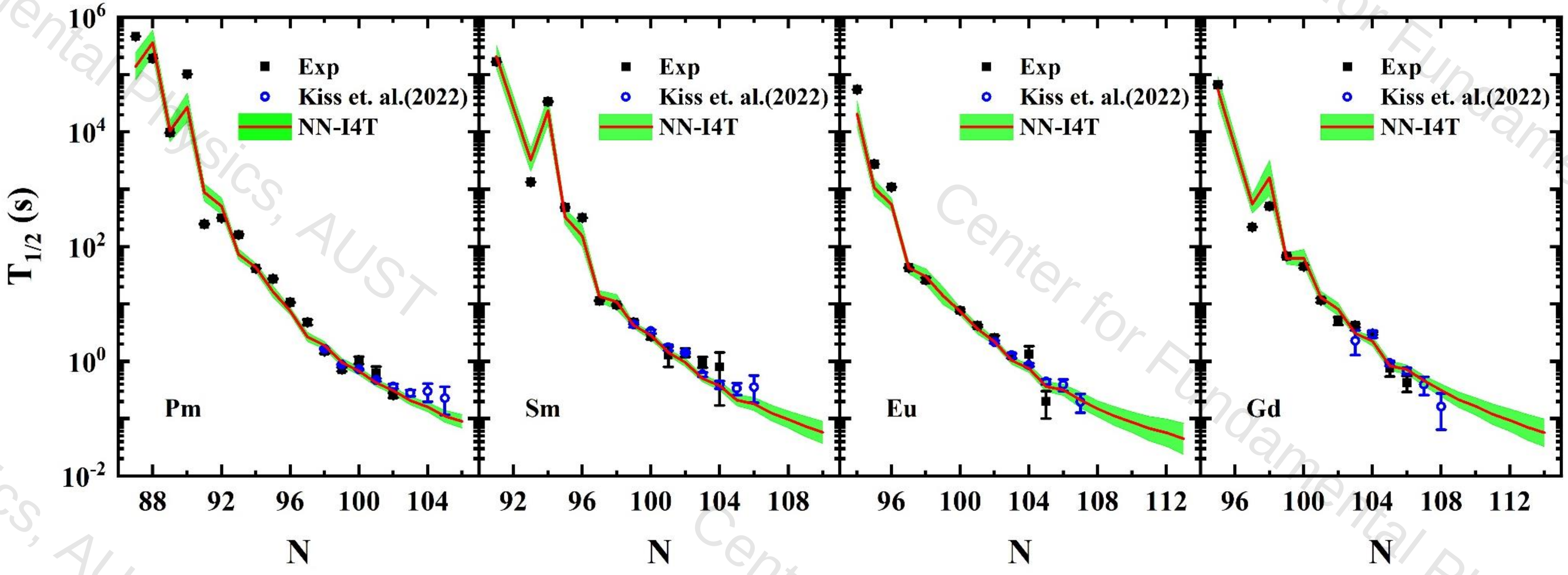


Fig. 3 Nuclear β-decay half-lives of Pm, Sm, Eu, and Gd isotopes. The new measurements of nuclear β-decay half-lives from Ref. [Kiss2022ApJ] are shown with open circles to test the extrapolation ability of the NN-I4T.

◆The prediction of the neural network is good agreement with the new measurements half-lives within the error range.

◆For the nuclei whose half-lives from NUBASE2020 and new measurements deviate with each other, such as ^{165,166}Sm and ^{168}Eu, the NN-I4T better reproduce the half-lives from new measurements.

Half-life predictions of NN and model averaging

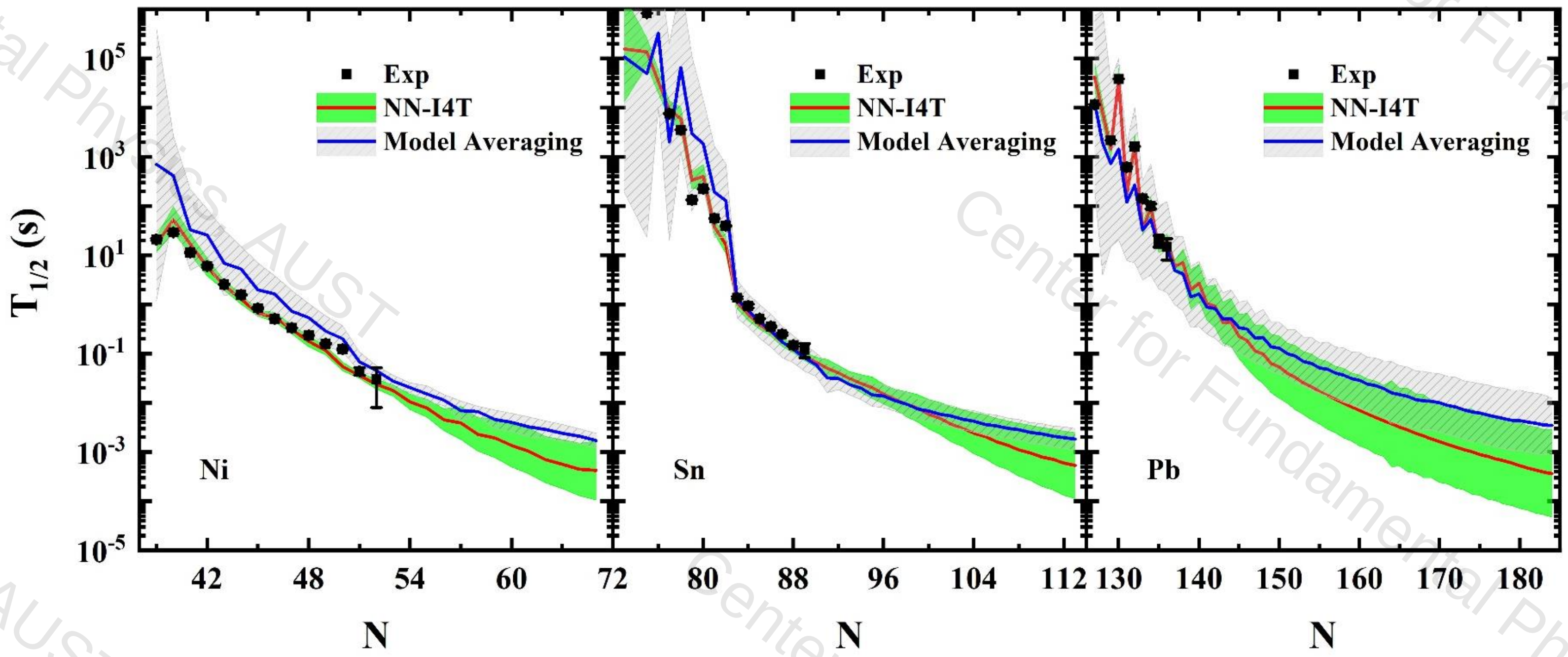
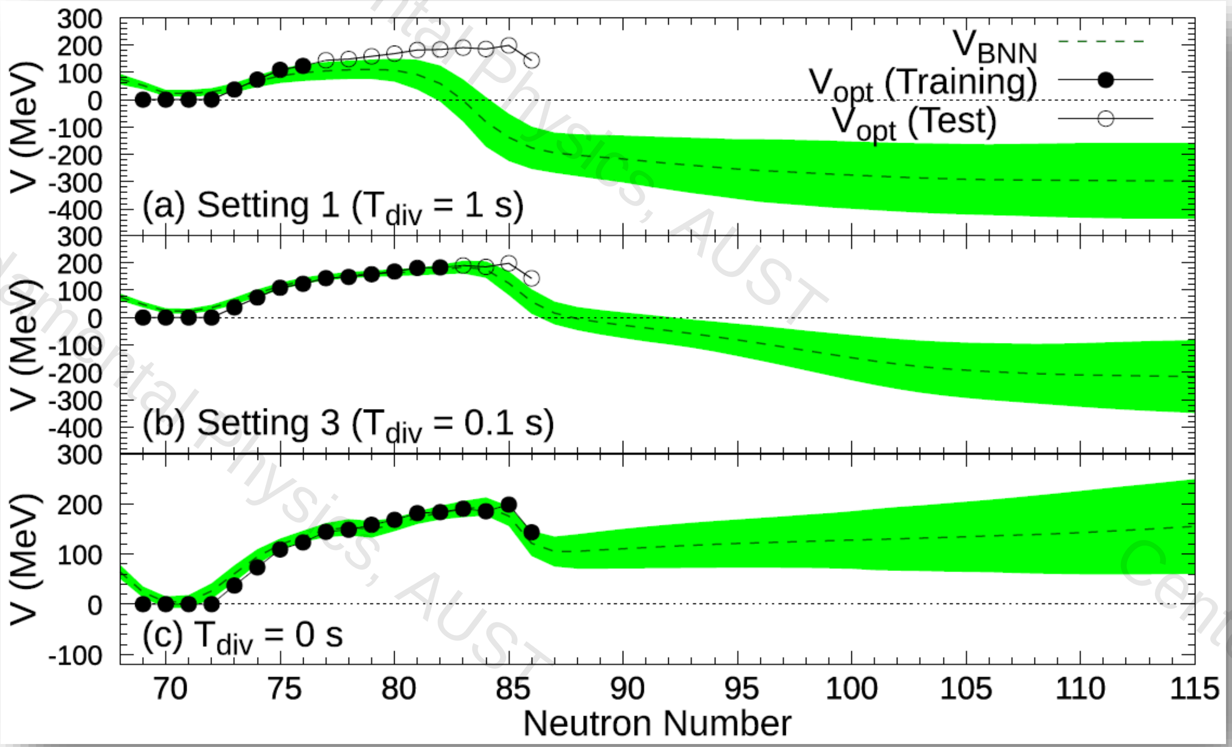


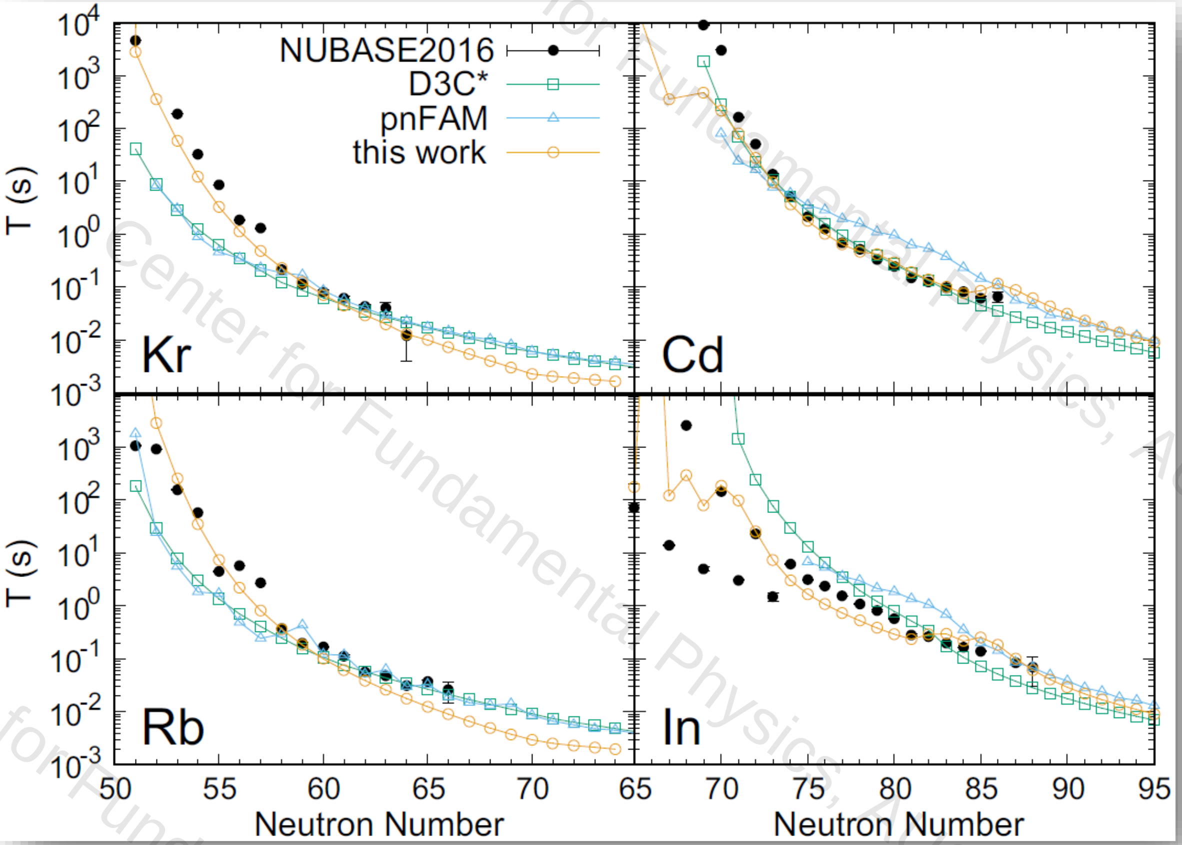
Fig. 4 Nuclear β-decay half-lives of Ni, Sn, and Pb isotopes. The uncertainties with NN-I4T and the averaging method are shown by the green hatched regions and slash hatched regions.

◆In the known region, the uncertainties of NN-I4T is generally smaller than that of the model averaging method.

◆ Near the very neutron-rich region, the uncertainties of NN-I4T increases rapidly and is even larger than that of the model averaging method.

β-decay half-lives from QRPA + machine learning





- The isoscalar pairing strengths determined by the BNN can reproduce experimental data with the same accuracy as other theoretical works. F. Minato *et al.*, PRC 106, 024306 (2022)

Monte-Carlo simulations of β-decay half-lives

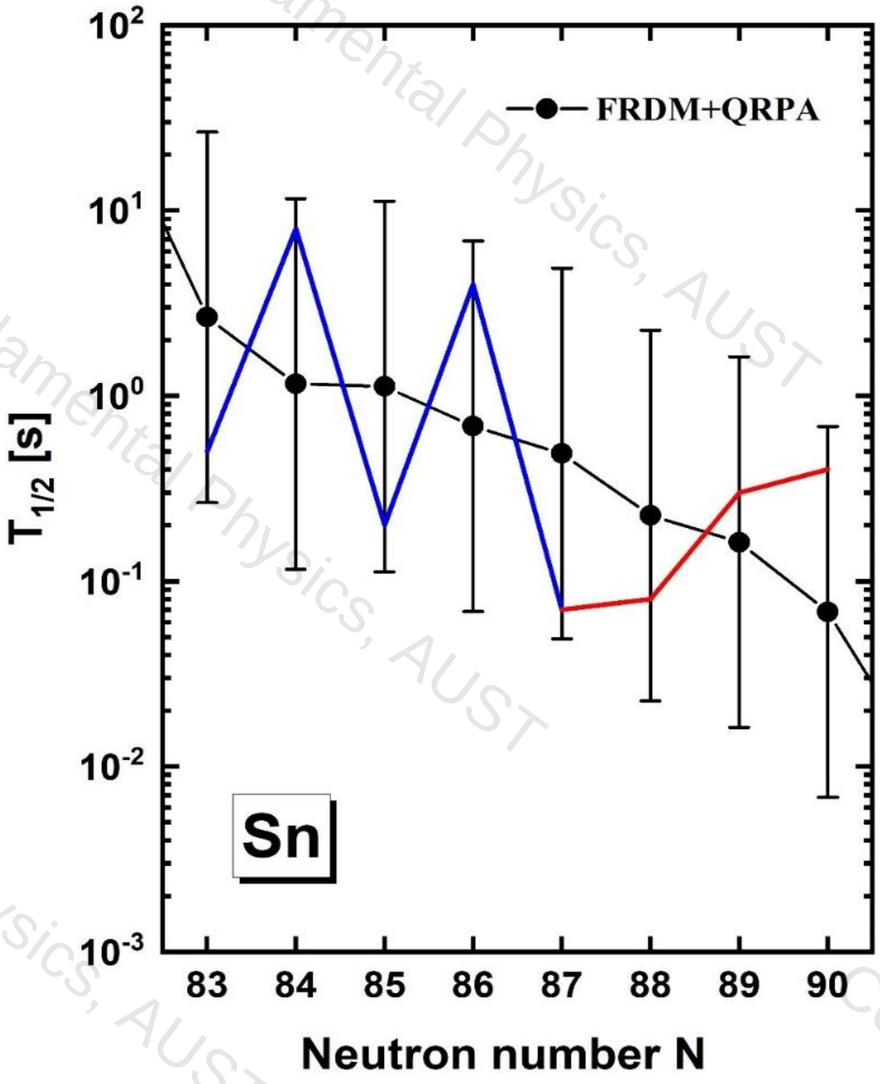


Fig: Diagram of Monte-Carlo simulation sampling.

- The β-decay half-lives: randomly produced by multiplying the factors that range from 0.1 to 10:
  - Opposite odd-even oscillations
  - Monotonically increases
- Nuclear β-decay models can avoid these non-physical trends

- investigate the effect of β-decay half-life uncertainties on the r-process simulations based on the predictions of various nuclear models.

r-process abundances

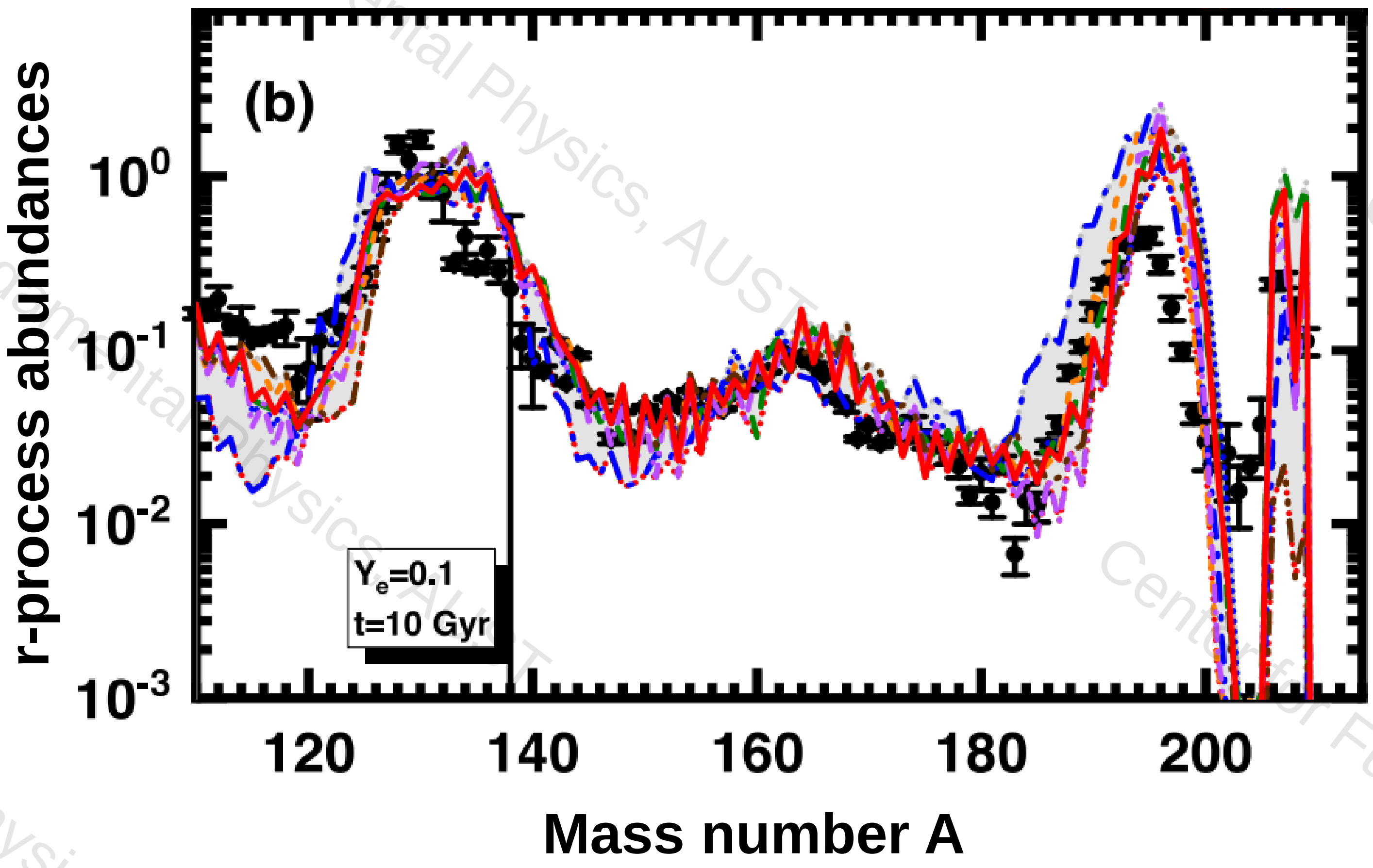


Fig: r-process abundances calculated with different β-decay half-life predictions at the moment of 10 Gyr.

- The uncertainties of the final r-process abundances from β-decay half-life predictions are about half an order of magnitude, which are much smaller than the uncertainties in Monte Carlo analysis.

J. Chen *et al.*, ApJ 943, 102 (2023)