Ripples of the QCD critical point

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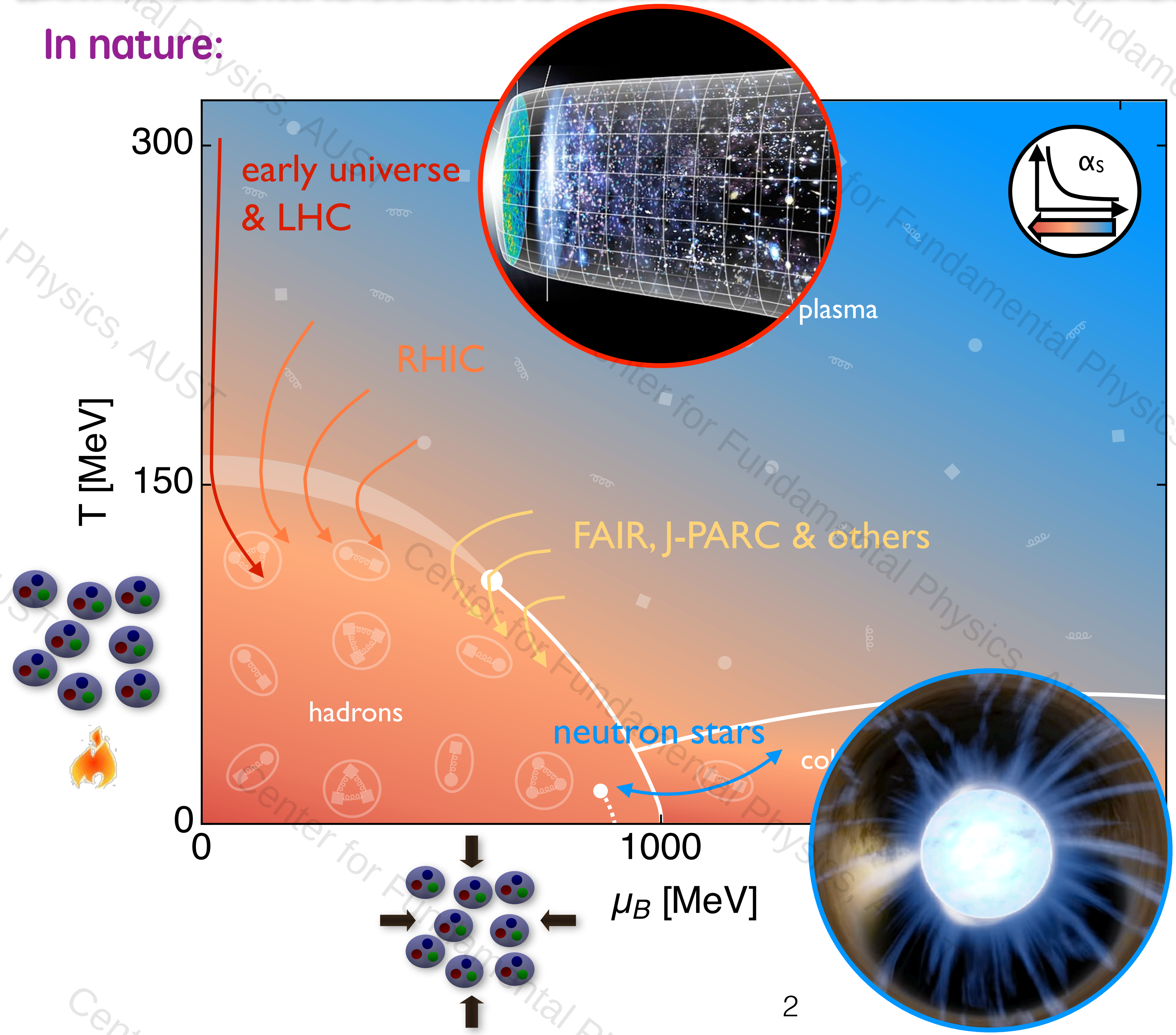
安徽理工大学，淮南，2024年4月26-30日

Based on :
WF, Xiaofeng Luo, Jan M. Pawlowski, Fabian Rennecke, Shi Yin, arXiv: 2308.15508;
Braun, Chen, WF, Gao, Huang, Ihssen, Pawlowski, Rennecke, Sattler, Tan, Wen, and Yin, arXiv:2310.19853;
WF, Chuang Huang, Jan M. Pawlowski, Yang-yang Tan, arXiv:2401.07638;
WF, Jan M. Pawlowski, Robert D. Pisarski, Fabian Rennecke, Rui Wen, and Shi Yin, in preparation.

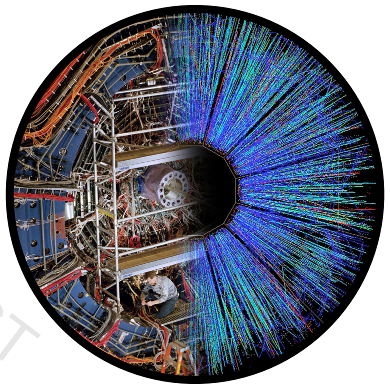
QCD phase diagram

In nature:

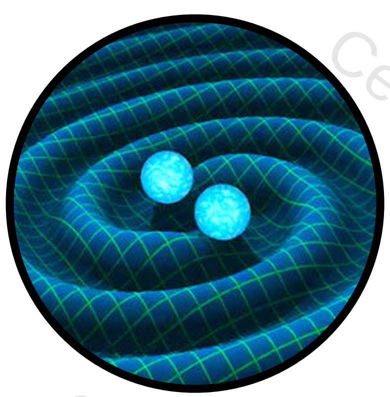


Experiments:

heavy-ion collisions

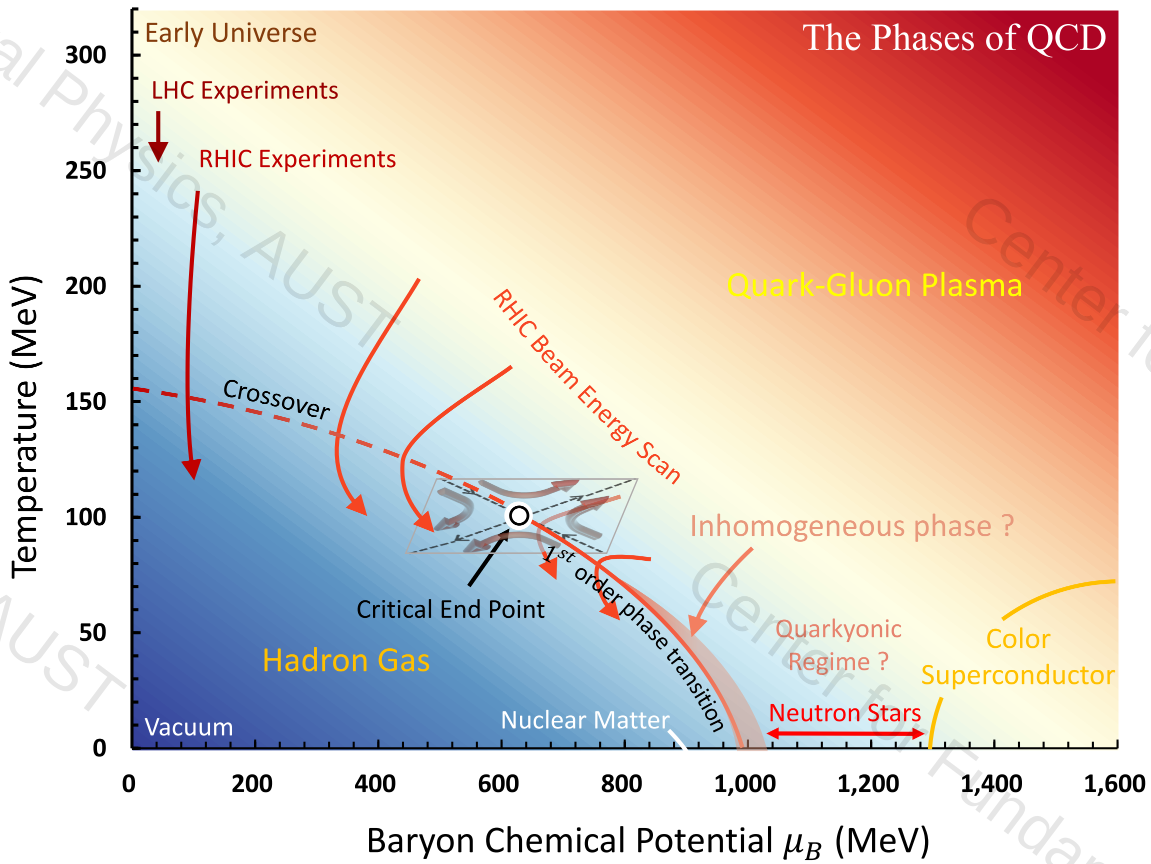


gravitational waves

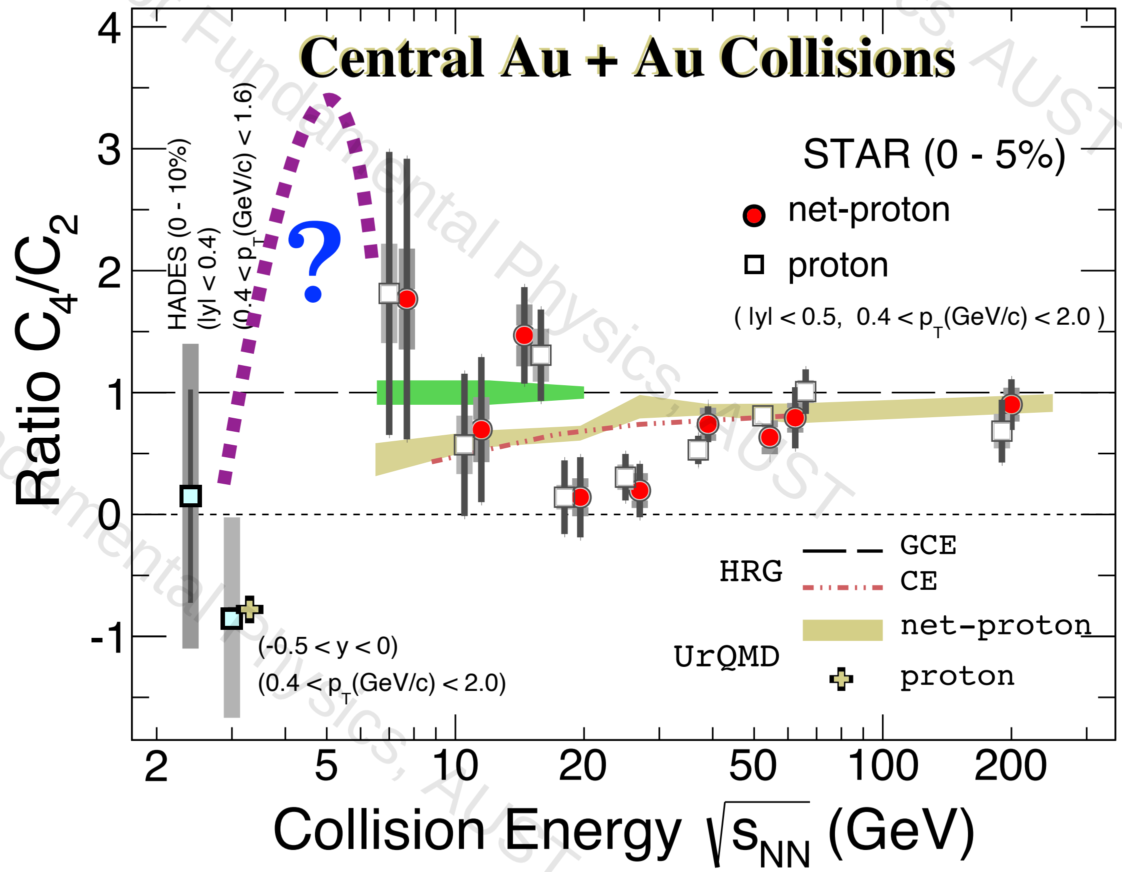


QCD critical point

QCD phase diagram



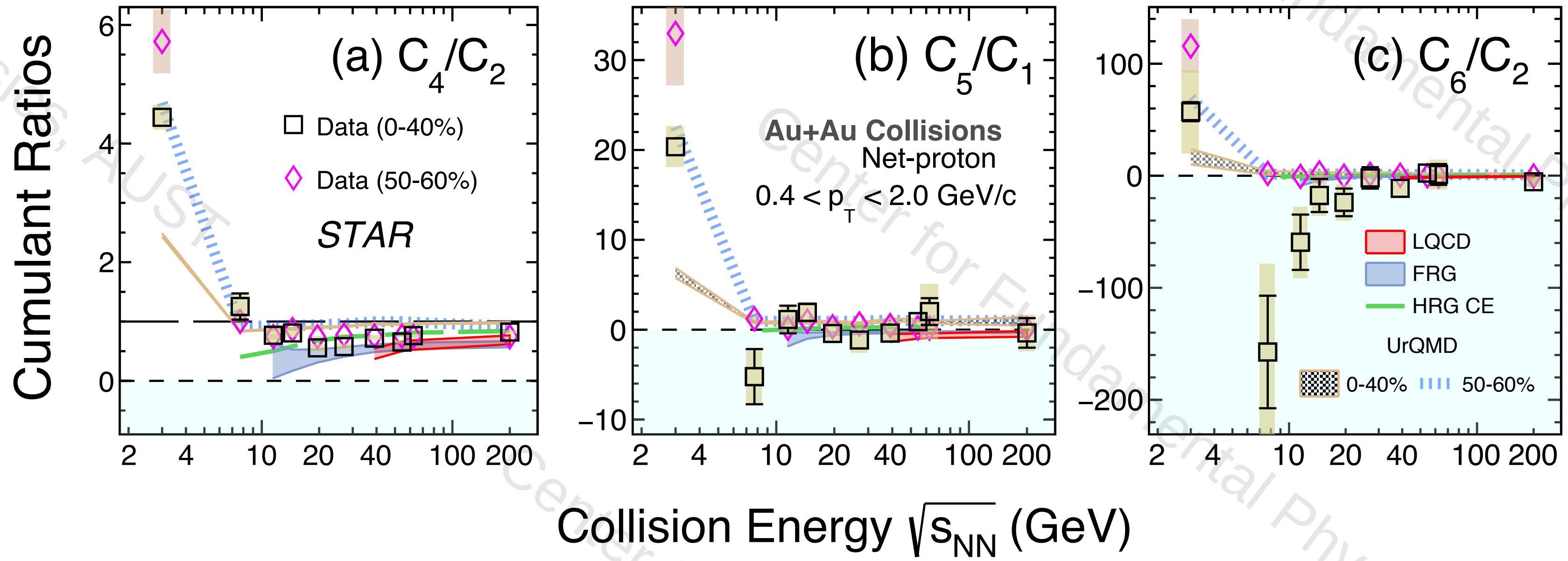
Fluctuations measured by STAR

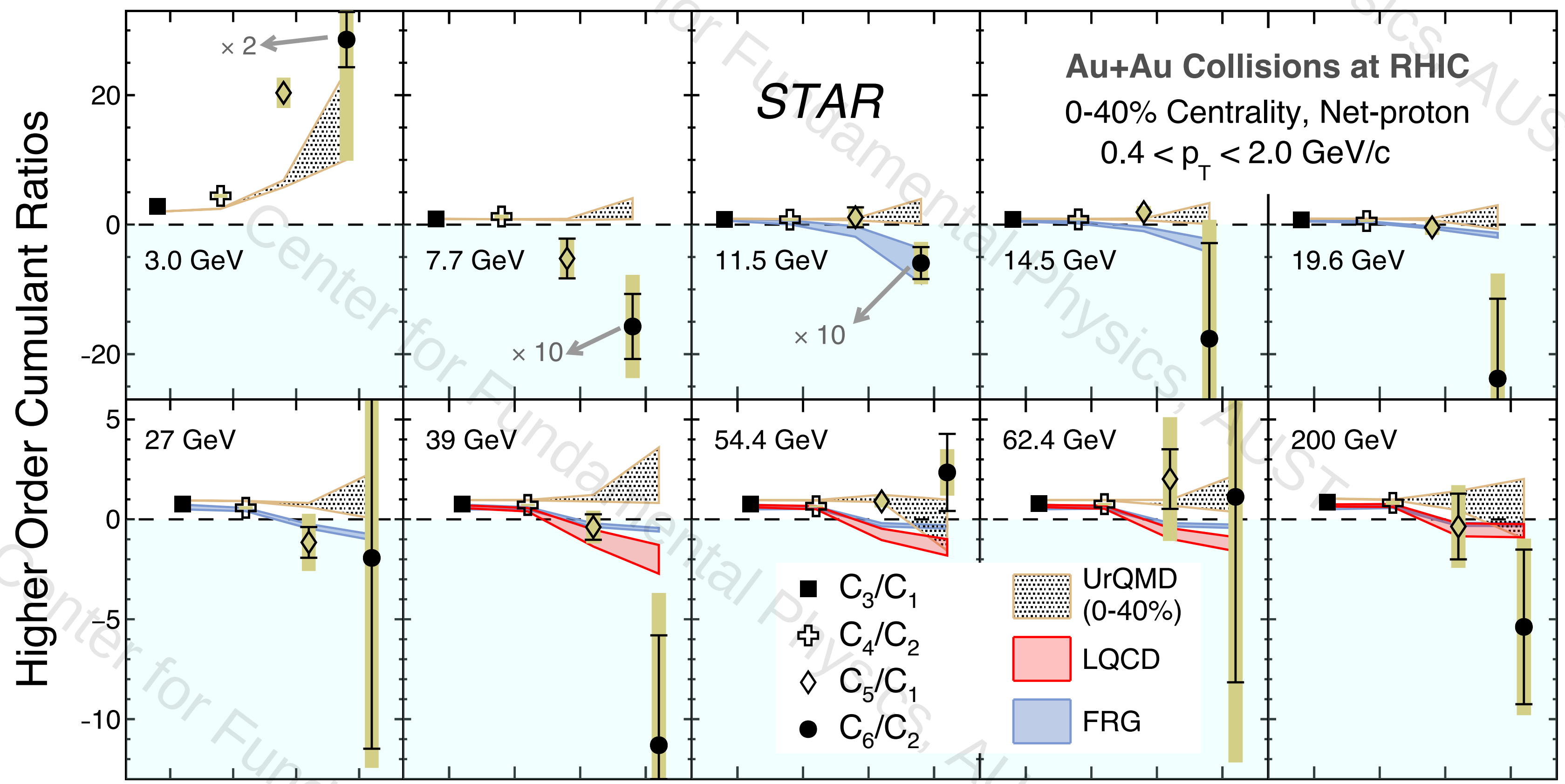


J. Adam *et al.* (STAR), *PRL* 126 (2021), 092301;
M. Abdallah *et al.* (STAR), *PRC* 104 (2021), 024902;
M. Abdallah *et al.* (STAR), *PRL* 128 (2022) 20, 202303

- The non-monotonicity of the kurtosis is observed with 3.1σ significance.
- Is there a "peak" structure in the regime of low colliding energy?

Hyper-order fluctuations





STAR: B. Aboona *et al.*, *PRL* 130 (2023), 082301, arXiv: 2207.09837

Outline

- Introduction
- Brief review about fRG
- Baryon number fluctuations and ripples of the QCD critical point
- Critical region and its size in QCD
- Moat regime in QCD
- Summary

Functional renormalization group

Functional integral with an IR regulator

$$Z_k[J] = \int (\mathscr{D}\hat{\Phi}) \exp\Big\{ -S[\hat{\Phi}] - \Delta S_k[\hat{\Phi}] + J^a \hat{\Phi}_a \Big\}$$

$$W_k[J] = \ln Z_k[J]$$

regulator:

$$\Delta S_k[\varphi] = \frac{1}{2}\int \frac{d^4 q}{(2\pi)^4} \varphi(-q) R_k(q) \varphi(q)$$

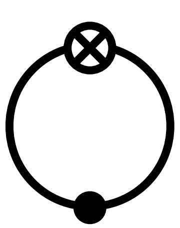
flow of the Schwinger function:

$$\partial_t W_k[J] = -\frac{1}{2}\mathrm{STr}\Big[\big(\partial_t R_k\big) G_k\Big] - \frac{1}{2}\Phi_a \partial_t R_k^{ab} \Phi_b$$

Legendre transformation:

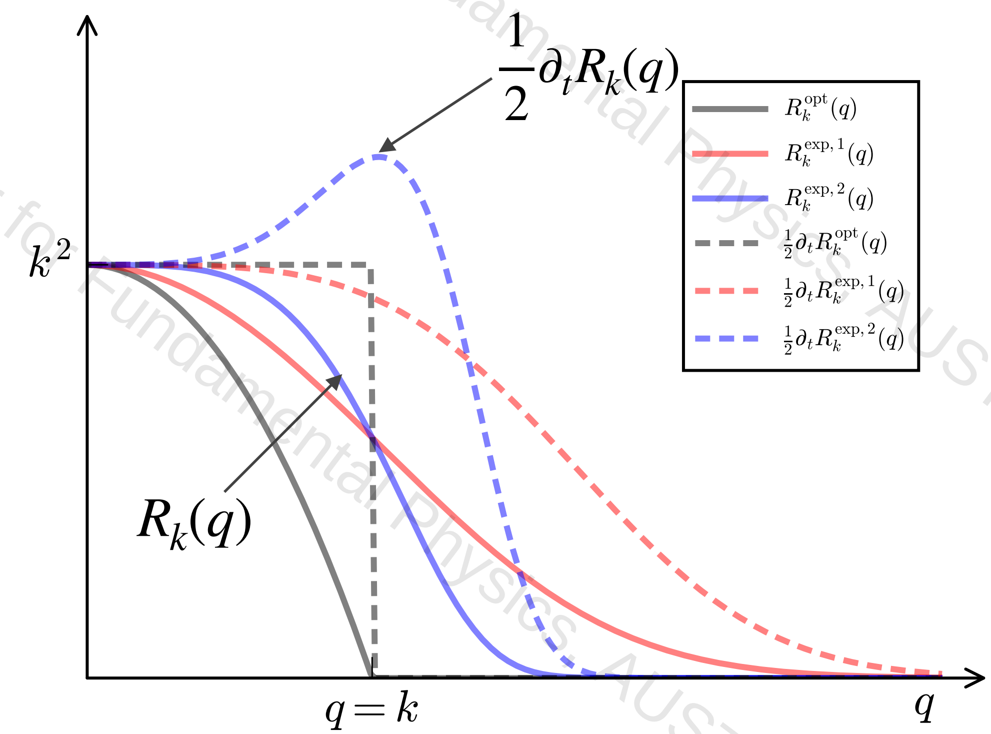
$$\Gamma_k[\Phi] = -W_k[J] + J^a \Phi_a - \Delta S_k[\Phi]$$

flow of the effective action:

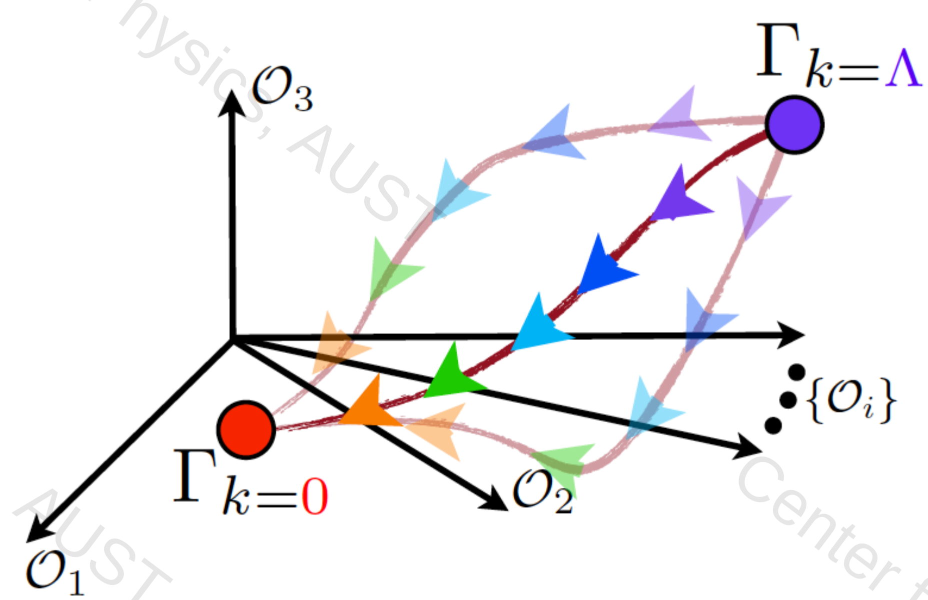
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\Big[\big(\partial_t R_k\big) G_k\Big] = \frac{1}{2}$$ 

Wetterich equation

C. Wetterich, *PLB*, 301 (1993) 90



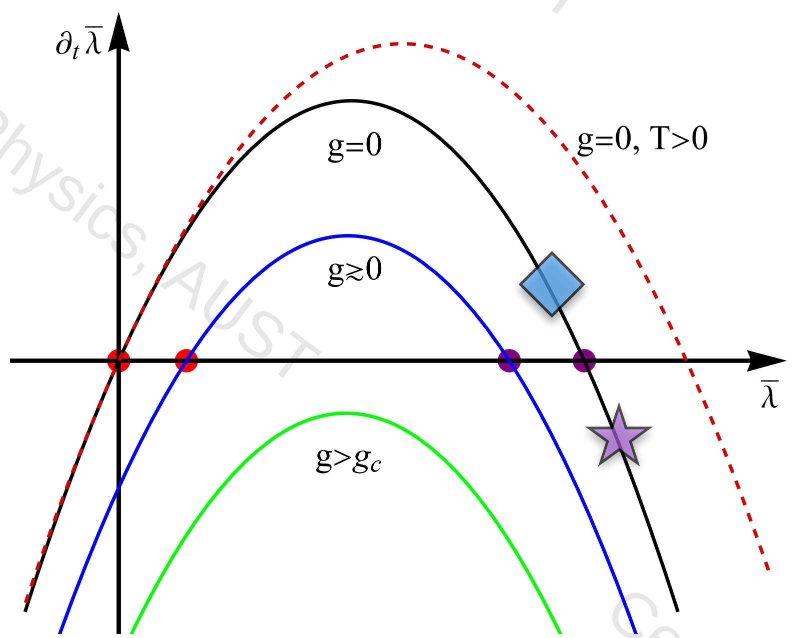
$$G_{k,ab} = \gamma^c{}_a \Big(\Gamma_k^{(2)}[\Phi] + \Delta S_k^{(2)}[\Phi]\Big)^{-1}_{cb},$$



Review: WF, *CTP* 74 (2022) 097304, arXiv: 2205.00468 [hep-ph]

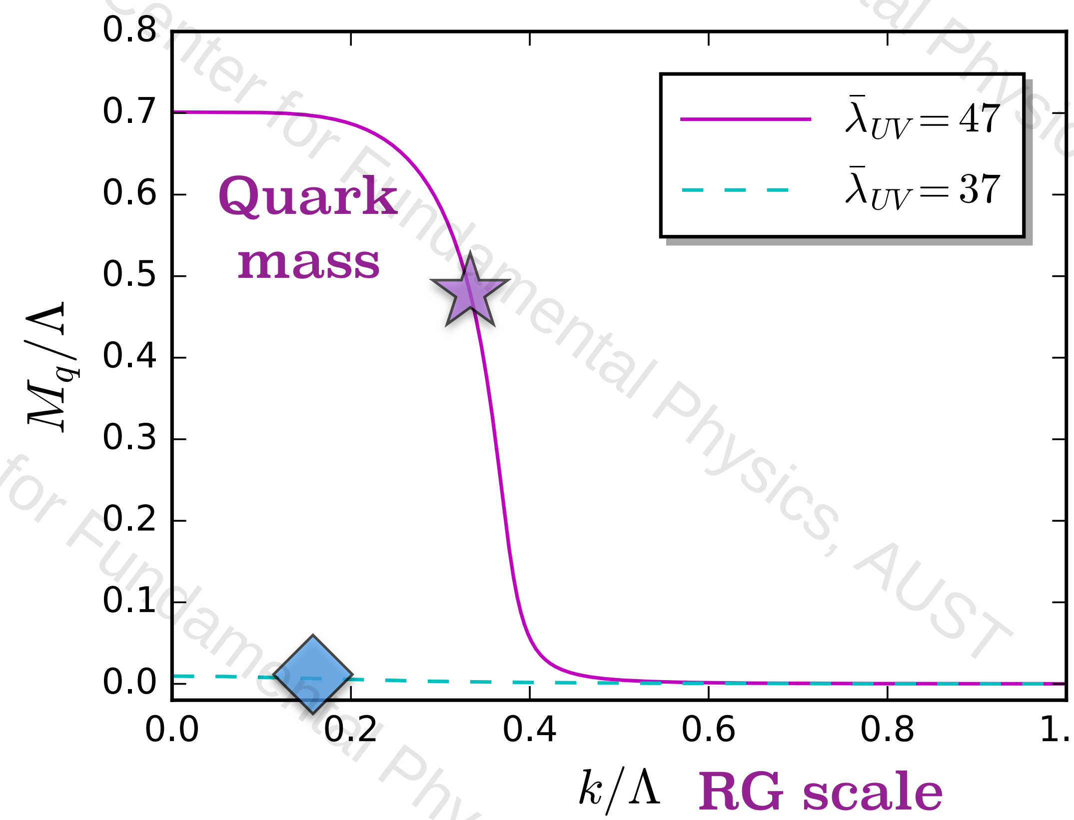
Chiral symmetry breaking in RG

- **β function of 4-quark coupling:**

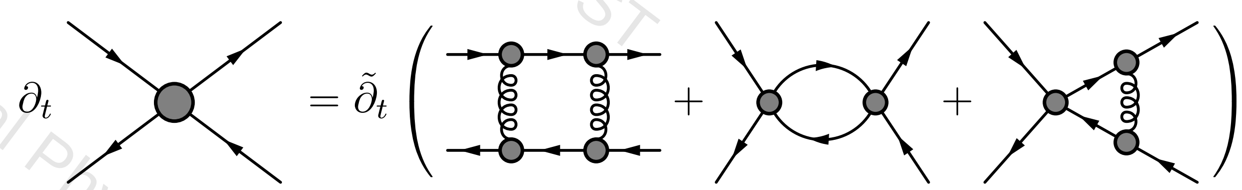


Braun, Gies, *JHEP* 06 (2006) 024.

$$\partial_t \bar{\lambda} = (d-2)\bar{\lambda} - a\bar{\lambda}^2 - b\bar{\lambda}g^2 - cg^4\,,$$

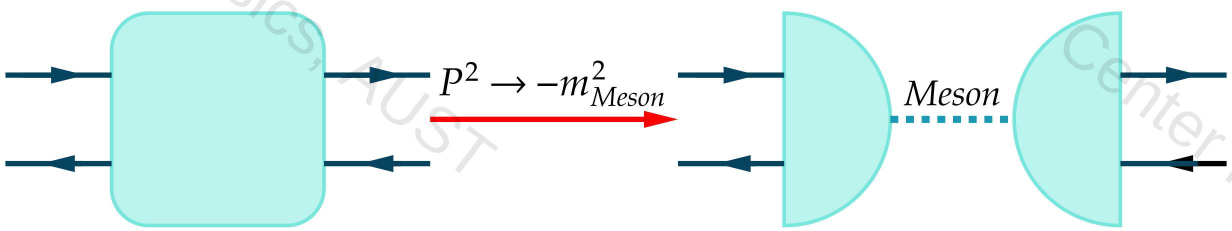


WF, Huang, Pawlowski, Tan, *SciPost Phys.* 14 (2023) 069, arXiv:2209.13120



Bound states in RG

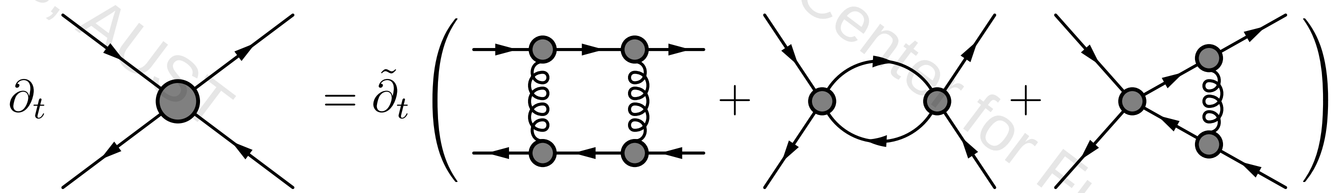
- **Bound states encoded in n-point correlation functions:**



$$\partial_t \lambda_{\pi,k}(P^2) = C_k(P^2)\lambda^2_{\pi,k}(P^2) + A_k(P^2),$$

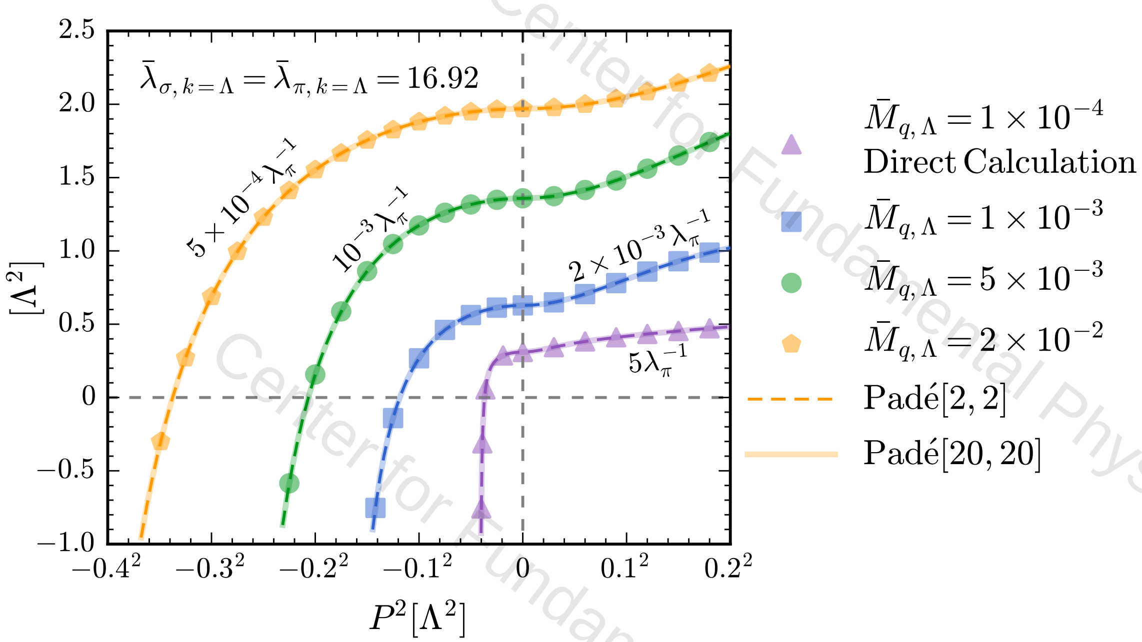
$$\lambda_{\pi,k=0}(P^2) = \frac{\lambda_{\pi,k=\Lambda}}{1-\lambda_{\pi,k=\Lambda}\int_\Lambda^0 C_k(P^2)\frac{dk}{k}},$$

- **Flow equation of 4-quark interaction:**



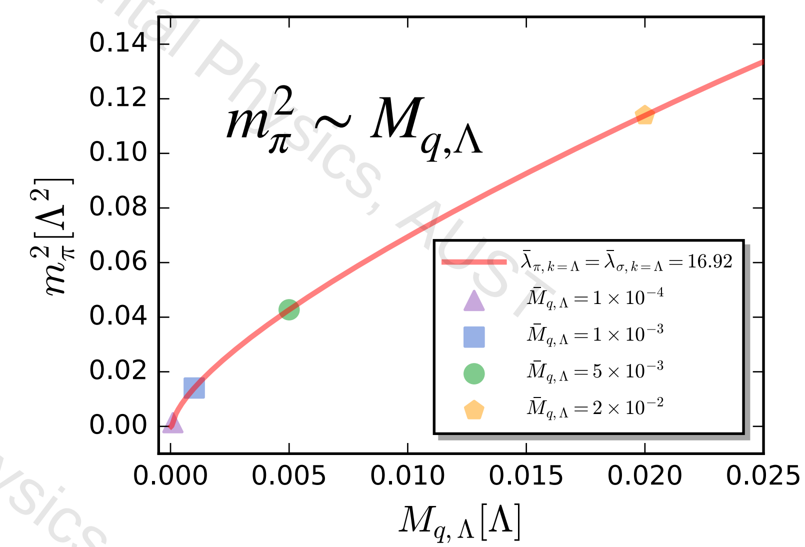
Note: playing the same role as the Bethe-Salpeter equation.

$$\frac{1}{\lambda_{\pi,k=0}}$$

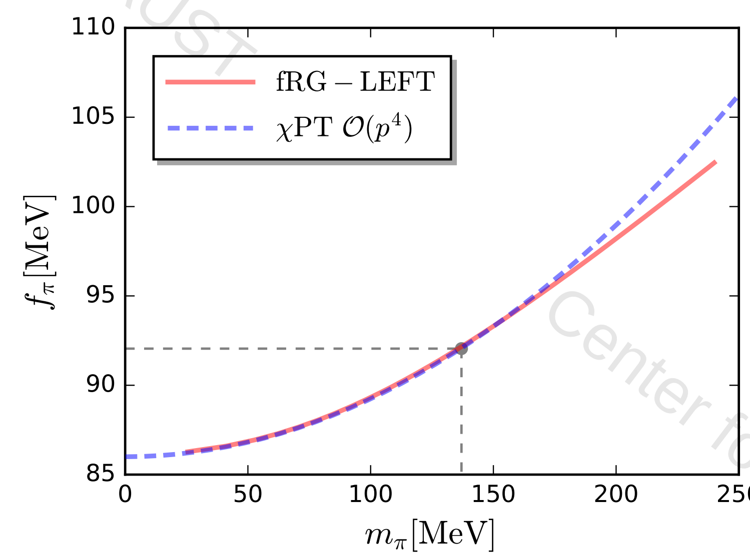


WF, Huang, Pawlowski, Tan, *SciPost Phys.* 14 (2023) 069, arXiv:2209.13120

Gell-Mann--Oakes--Renner relation



Pion decay constant



WF, Huang, Pawlowski, Tan, arXiv:2401.07638

Quasi-PDA of pion

- Bethe-Salpeter amplitude (unamputated):

$$\chi_\pi(k;P) = S(k_+)\Gamma(k;P)S(k_-)$$

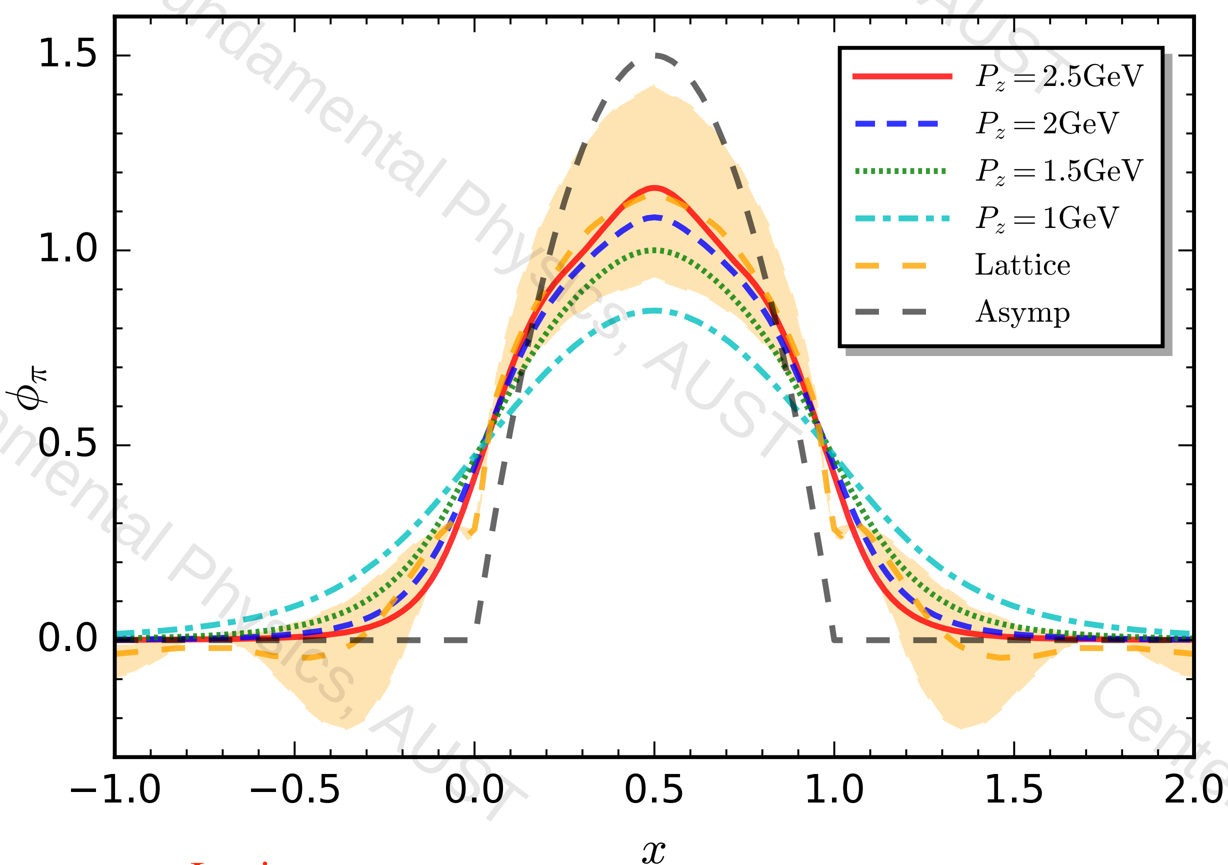
with $P = (iE_P, 0, 0, P_z)$, $P^2 = -m_\pi^2$ and $k_\pm = k \pm P/2$

- Quasi parton distribution amplitude (qPDA) reads

$$\tilde{\varphi}_\pi(x) = \frac{1}{f_\pi}\mathrm{Tr}_{\mathrm{CD}}\left[\int \frac{d^4k}{(2\pi)^4}\delta(\tilde{n}\cdot k_+ - x\tilde{n}\cdot P)\gamma_5\gamma\cdot\tilde{n}\chi_\pi(k;P)\right]$$

with $\tilde{n} = (0,0,0,1)$.

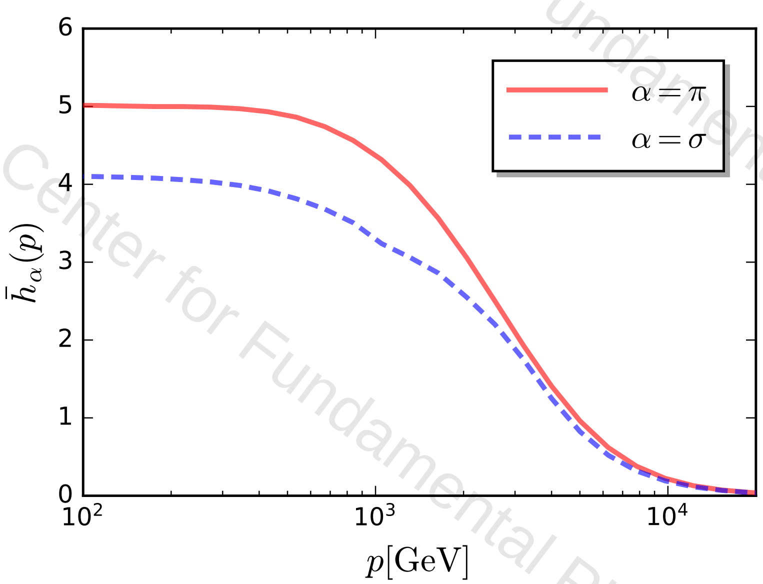
Quasi-PDA:



Lattice:
Zhang *et al.*, *PRD* 95 (2017) 094514, arXiv: 1702.00008;
Hua *et al.*, *PRL* 129 (2022) 132001, arXiv: 2201.09173

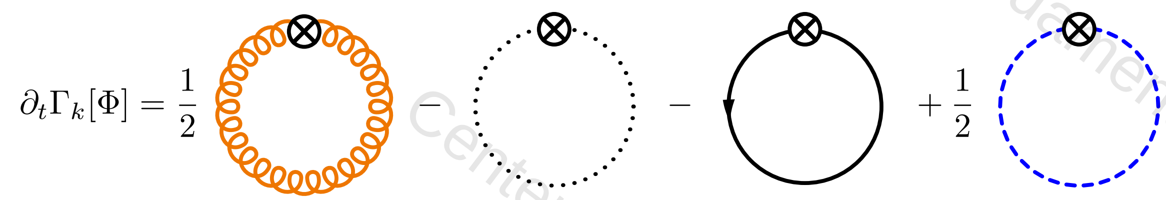
fRG:
Huang, Zhang, WF, Chang, in preparation

BS amplitude:

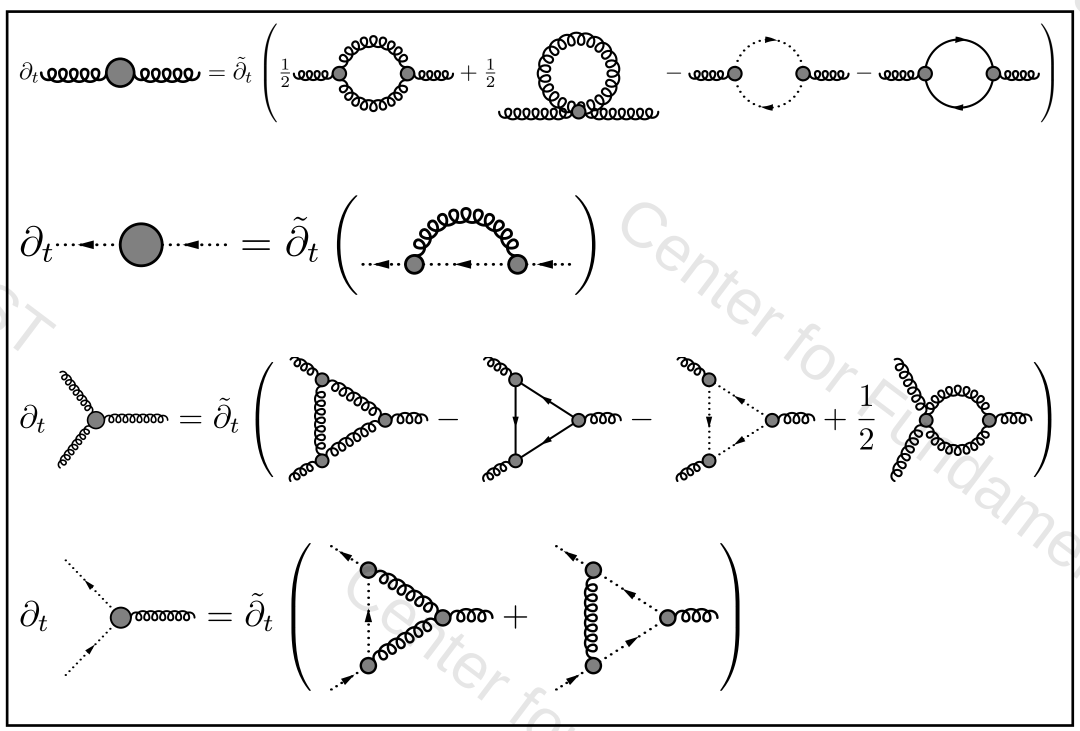


First-principles QCD within fRG

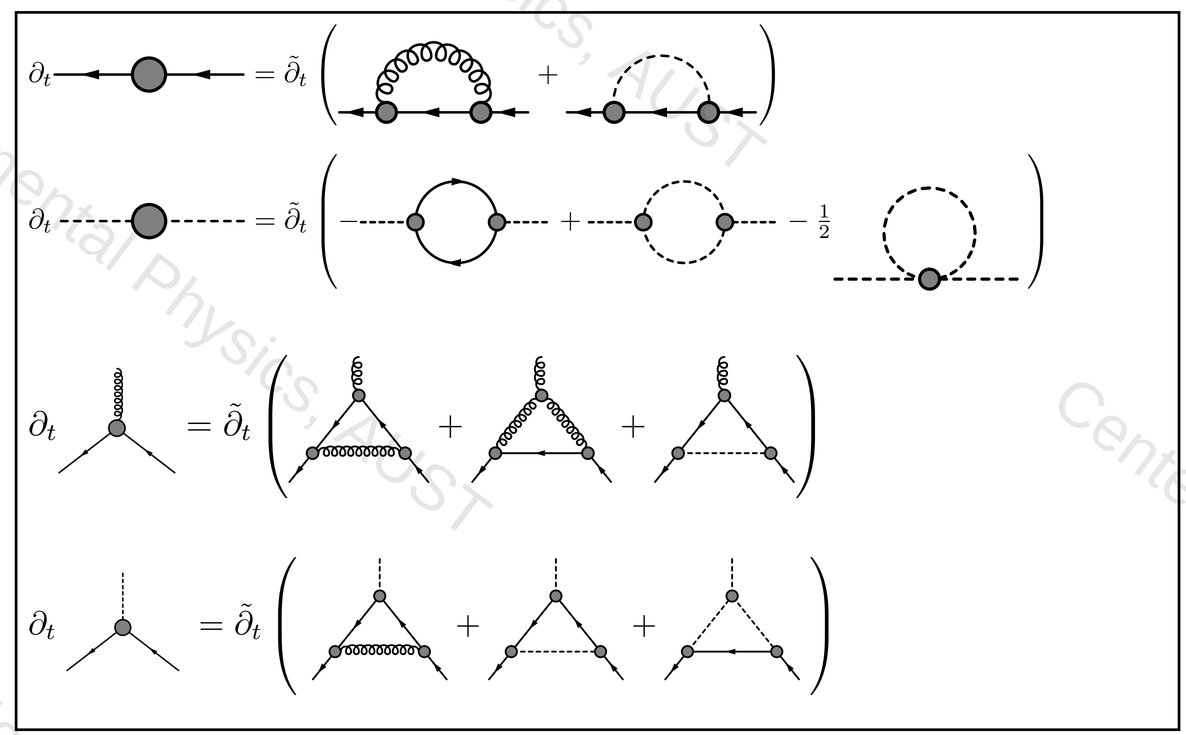
QCD flow equation:



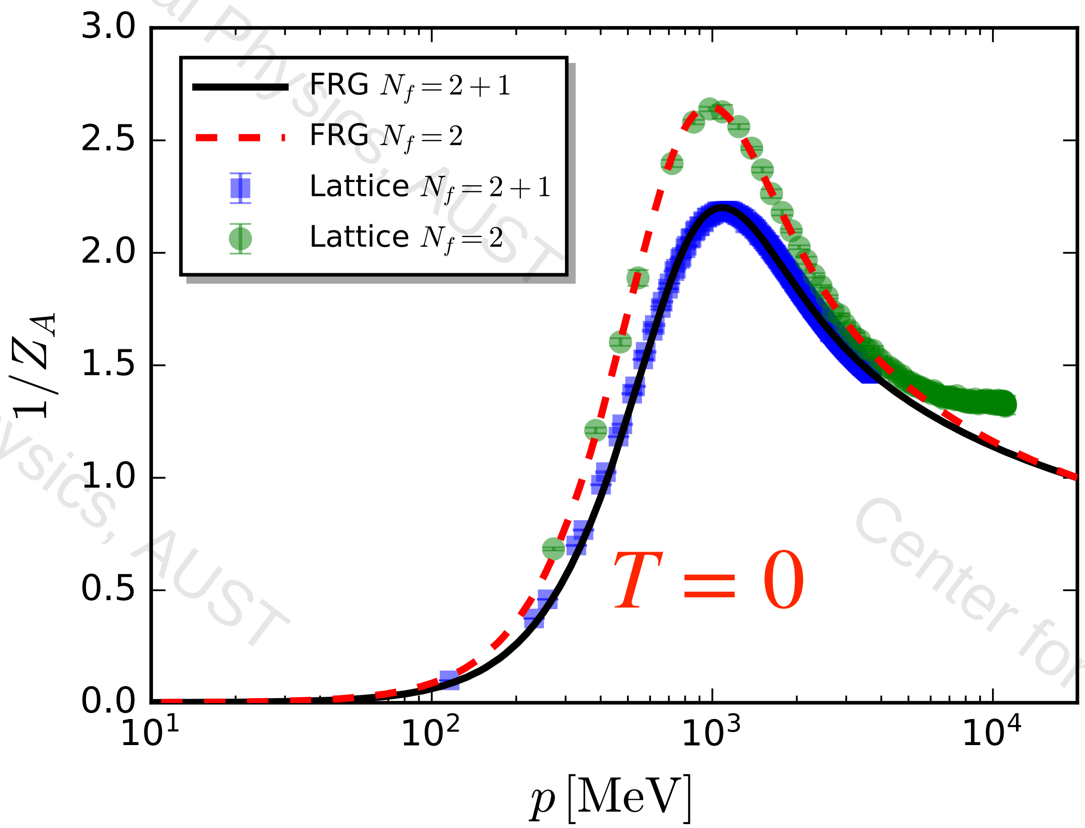
Glue sector:

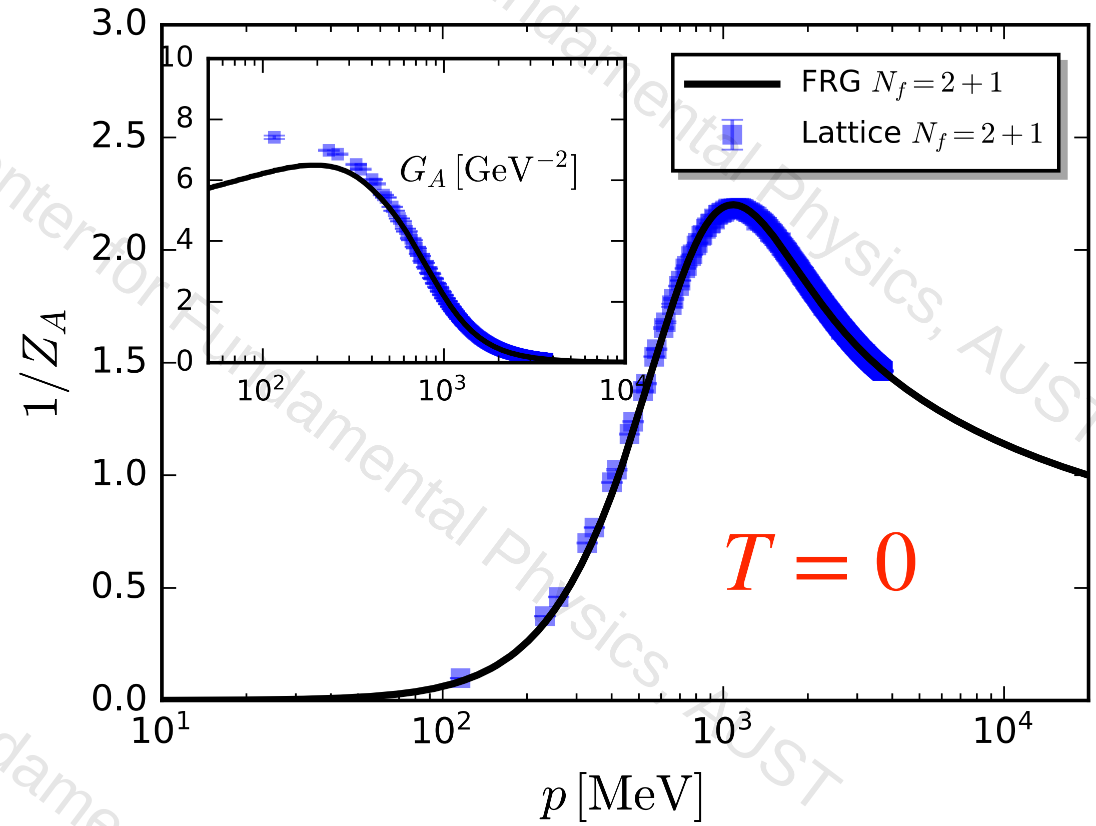


Matter sector:



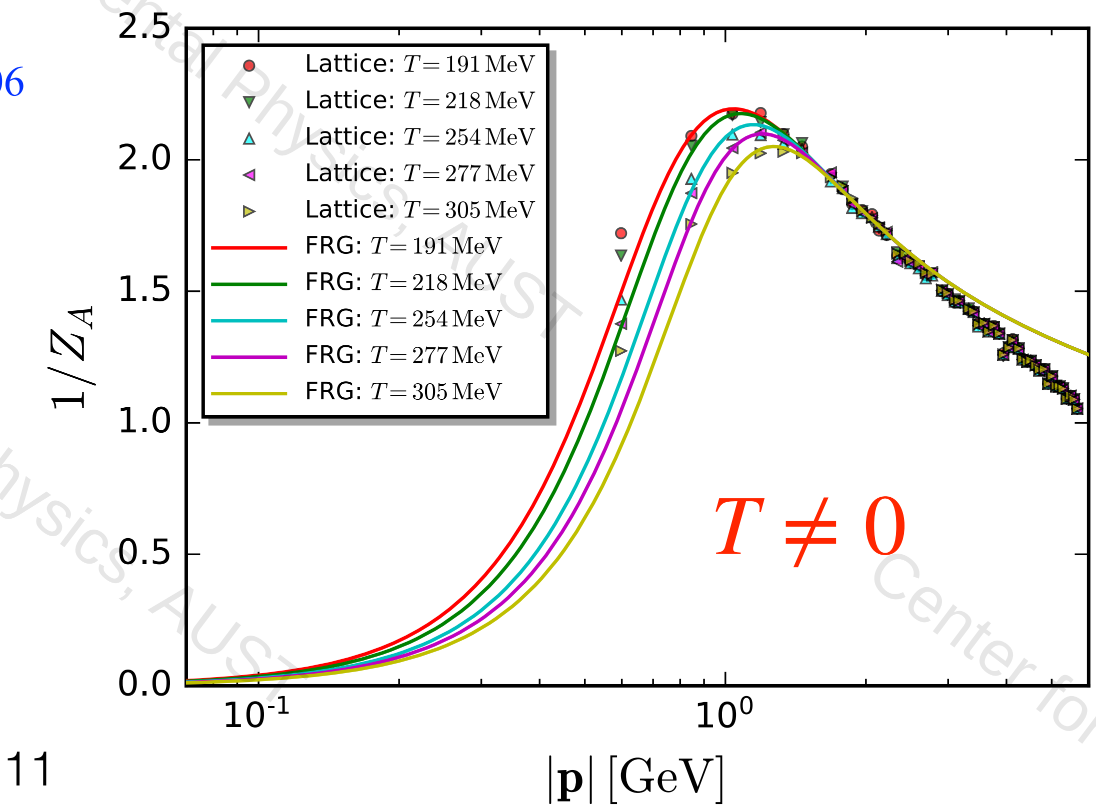
Gluon dressing functions



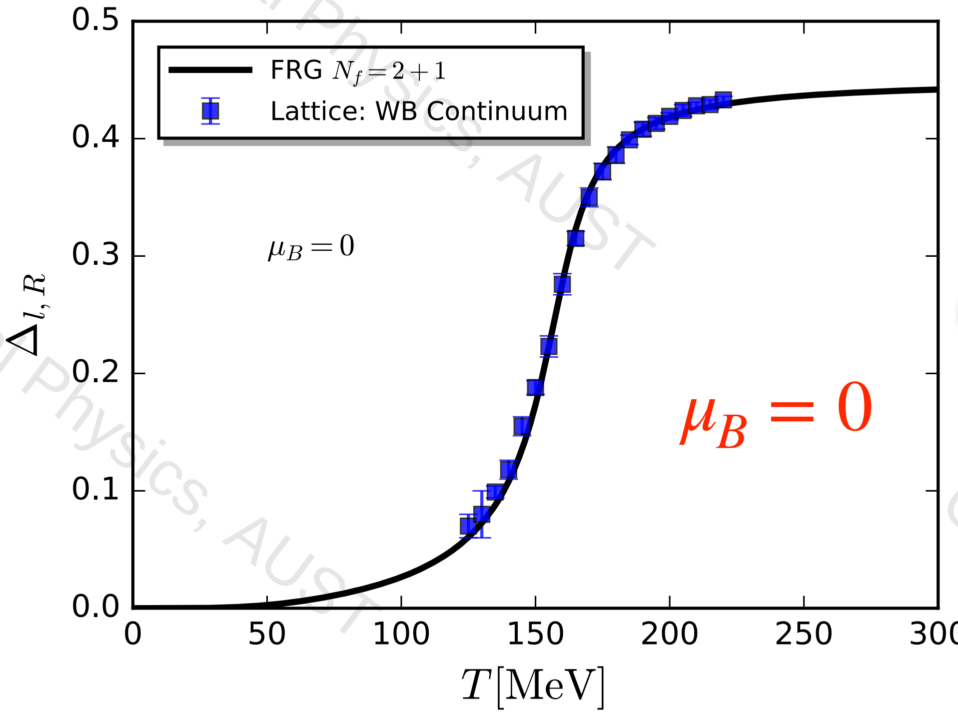


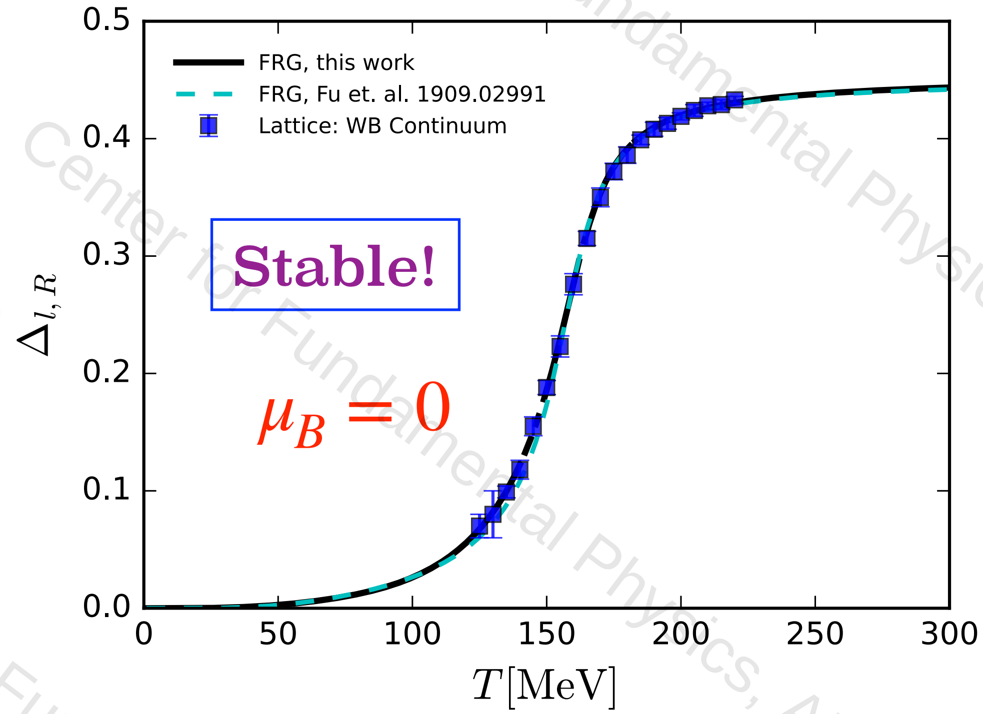
fRG $N_f = 2$: Cyrol, Mitter, Pawlowski,Strodthoff, *PRD* 97 (2018) 054006
Lattice $N_f = 2$: Sternbeck *et al.*, *PoS* (2012) LATTICE2012, 243
Lattice $N_f = 2 + 1$: Boucaud *et al.*, *PRD* 98 (2018) 114515

fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032



Renormalized light quark condensate



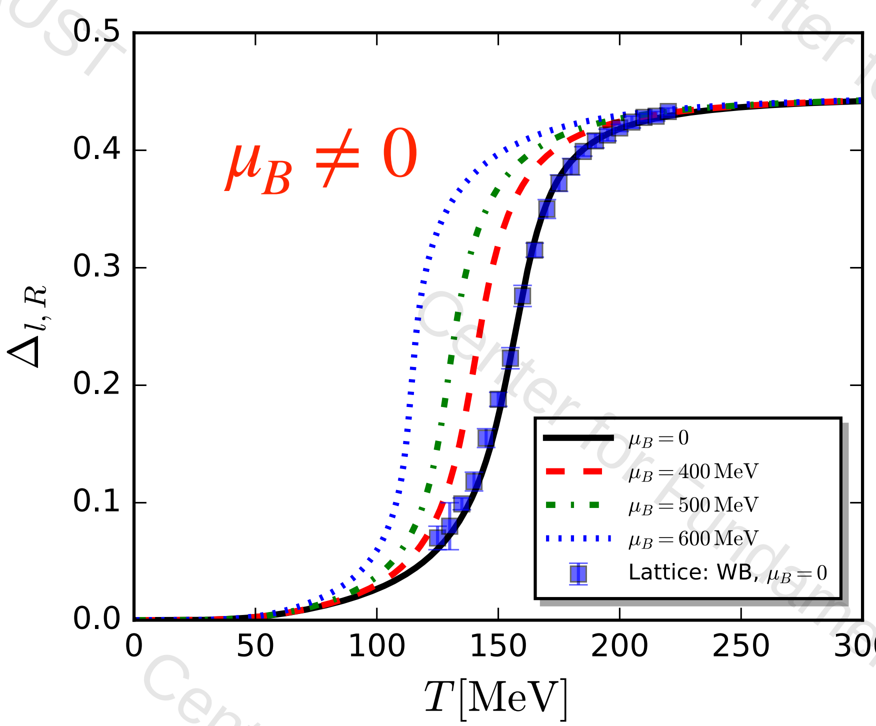


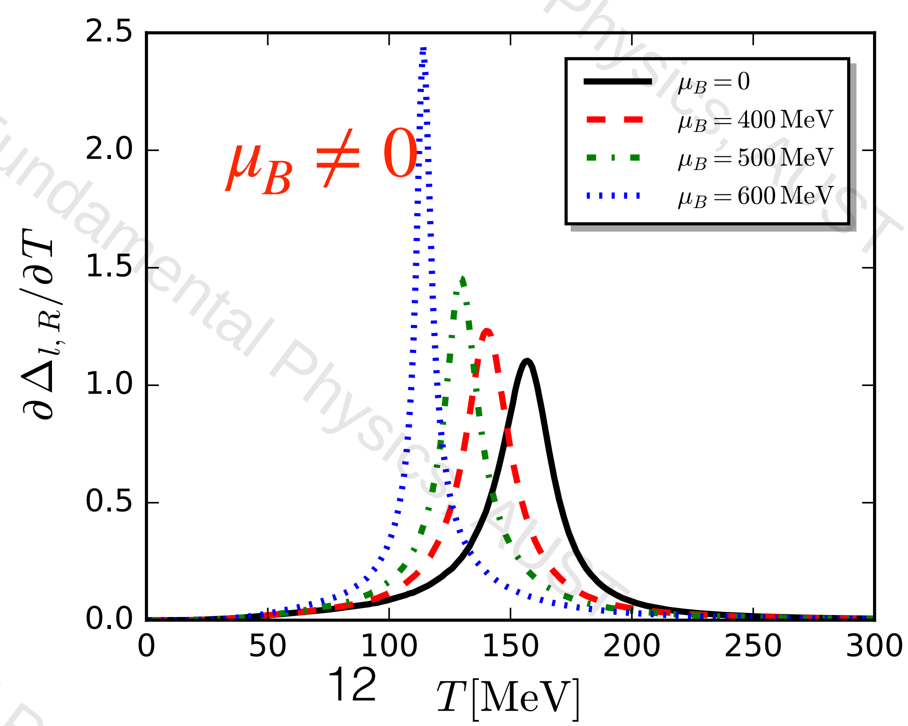
improved truncations for the sector of s quark and the full mesonic potential of $N_f = 2+1$.

Lattice: Borsanyi *et al.* (WB), *JHEP* 09 (2010) 073

fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

fRG: WF, Pawlowski, Wen, Yin, (2024) in preparation



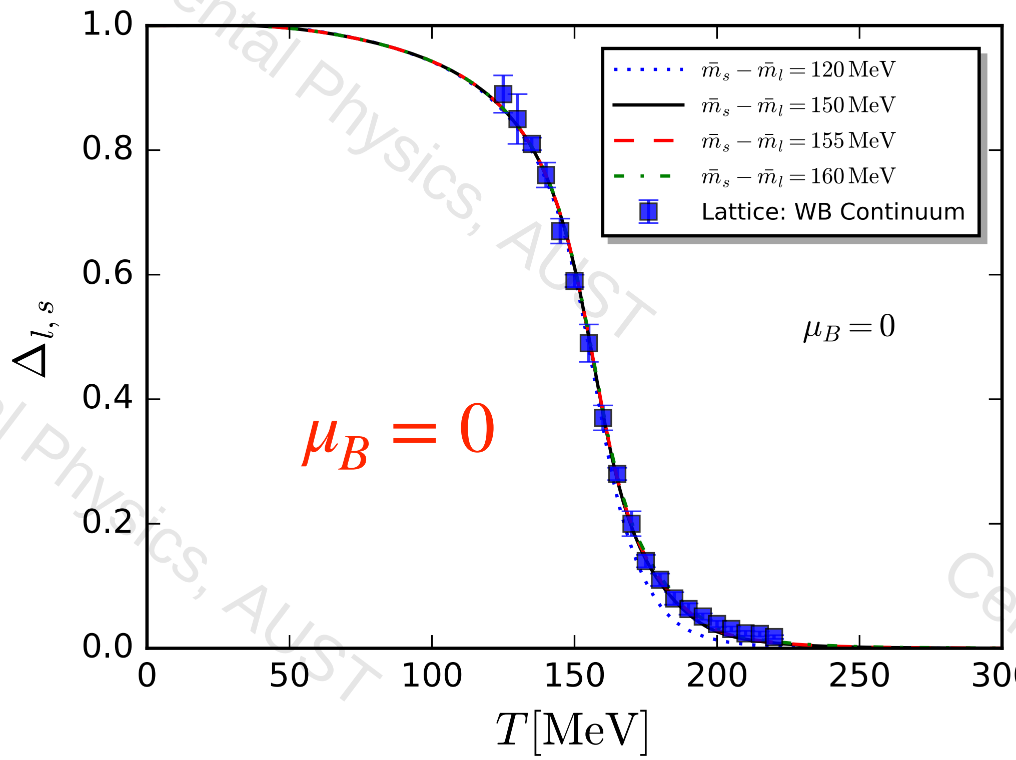


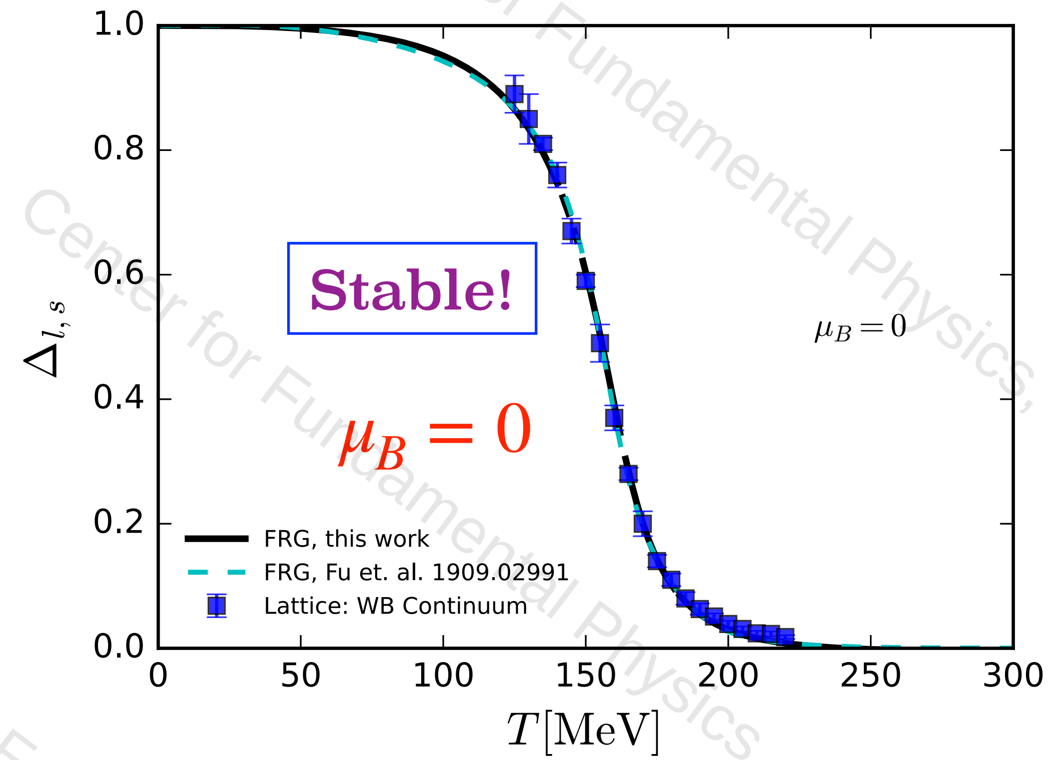
quark condensate:

$$\Delta_{q_i} \simeq -m^0_{q_i} T \sum_{n\in\mathbb{Z}} \int \frac{d^3q}{(2\pi)^3} \mathrm{tr}\, G_{q_i\bar{q}_i}(q)\,,$$

$$\Delta_{q_i,R} = \frac{1}{\mathcal{N}_R}\left[\Delta_{q_i}(T,\mu_q) - \Delta_{q_i}(0,0)\right]\,.$$

Other fermionic observables





improved truncations for the sector of s quark and the full mesonic potential of N_f = 2+1.

Lattice: Borsanyi *et al.* (WB), *JHEP* 09 (2010) 073

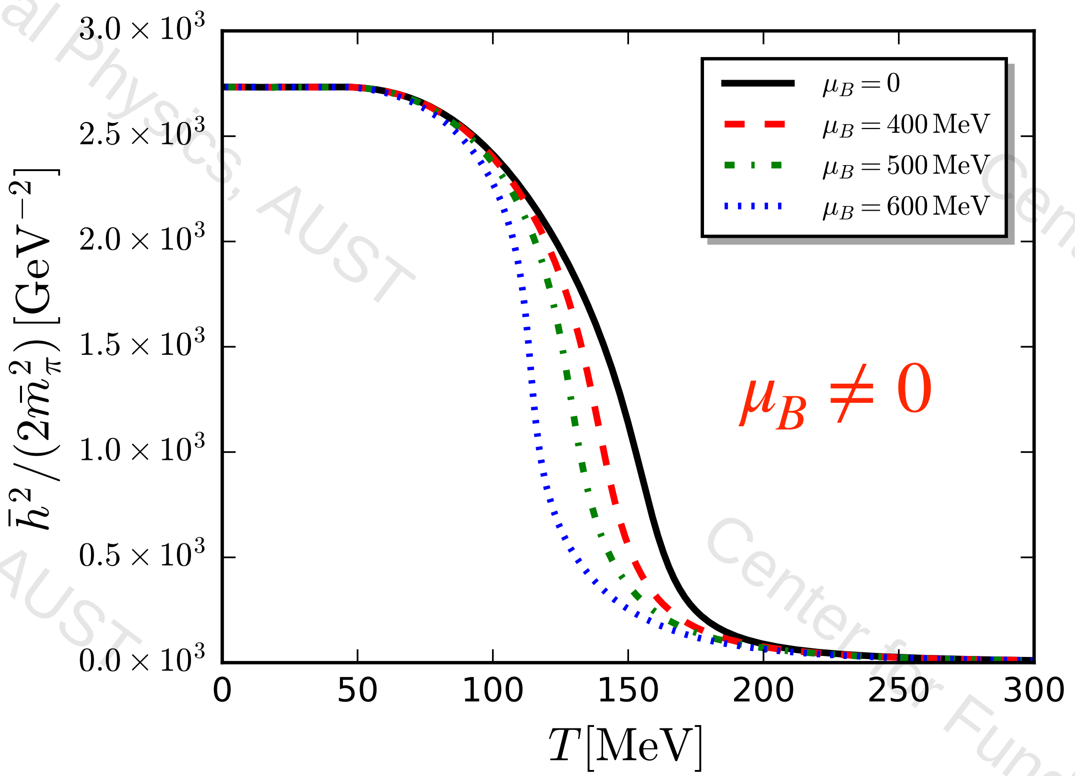
fRG: WF, Pawlowski, Wen, Yin, (2024) in preparation

fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

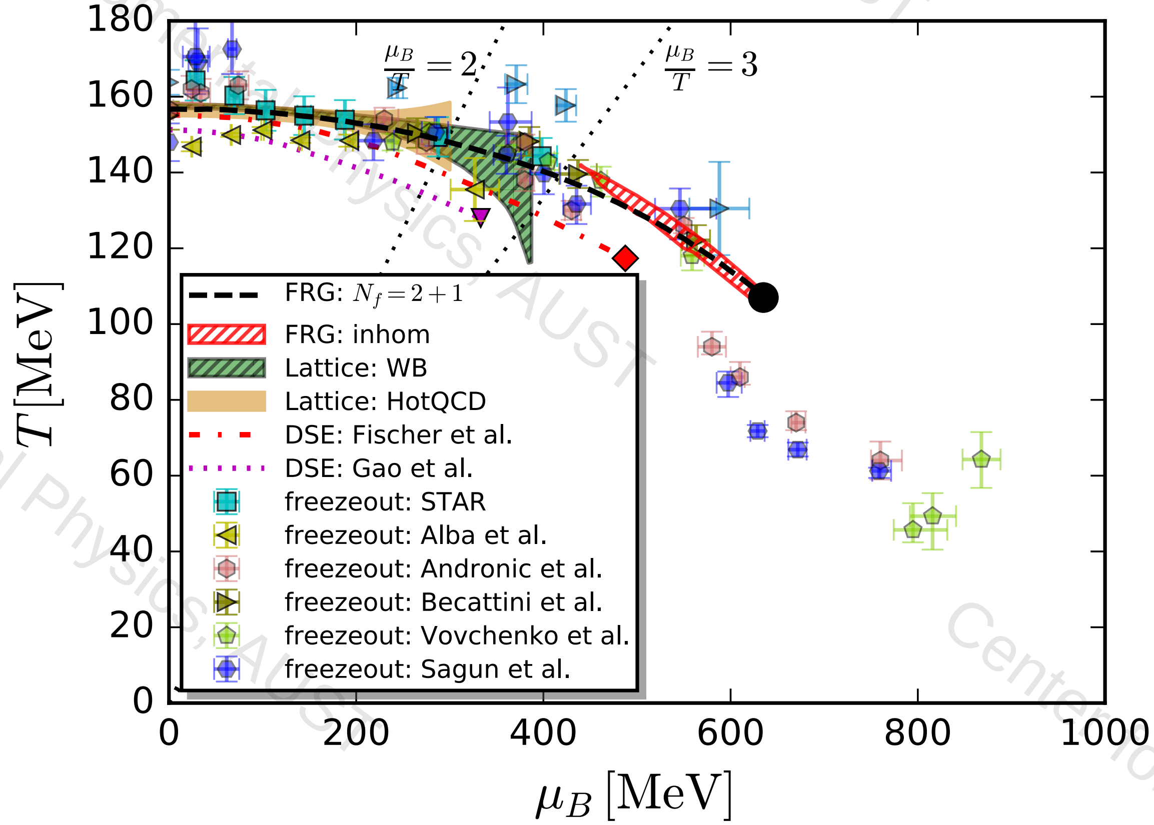
Reduced condensate:

$$\Delta_{l,s}(T,\mu_q) = \frac{\Delta_l(T,\mu_q) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(T,\mu_q)}{\Delta_l(0,0) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(0,0)}$$

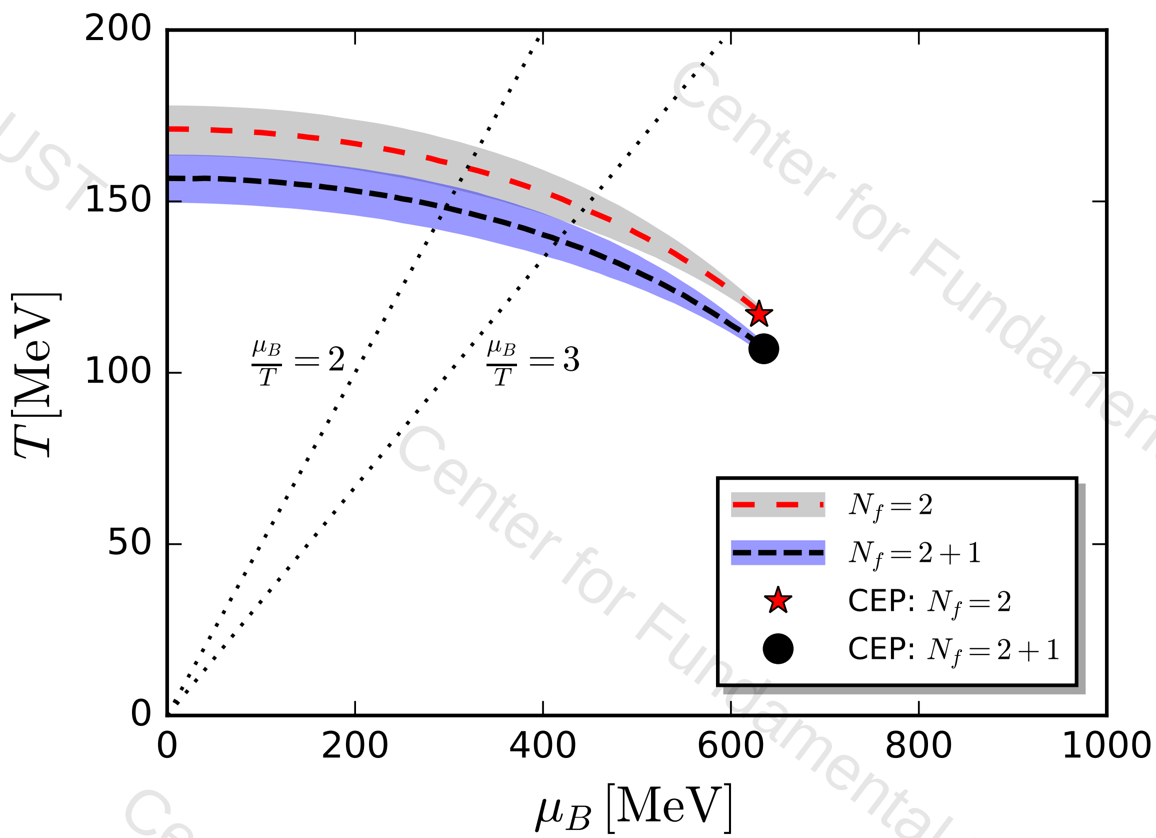
Effective four-quark coupling:



Phase boundary and curvature



fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032



CEP:

$$(T_{\mathrm{CEP}}, {\mu_B}_{\mathrm{CEP}})_{N_f=2+1} = (107\,\mathrm{MeV}, 635\,\mathrm{MeV})\,,$$

$$(T_{\mathrm{CEP}}, {\mu_B}_{\mathrm{CEP}})_{N_f=2} = (117\,\mathrm{MeV}, 630\,\mathrm{MeV})\,,$$

FRG curvature of the phase boundary:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa\left(\frac{\mu_B}{T_c}\right)^2 + \lambda\left(\frac{\mu_B}{T_c}\right)^4 + \cdots\,,$$

$$\kappa_{N_f=2+1} = 0.0142(2)$$

$$\kappa_{N_f=2} = 0.0176(1)$$

Lattice result:

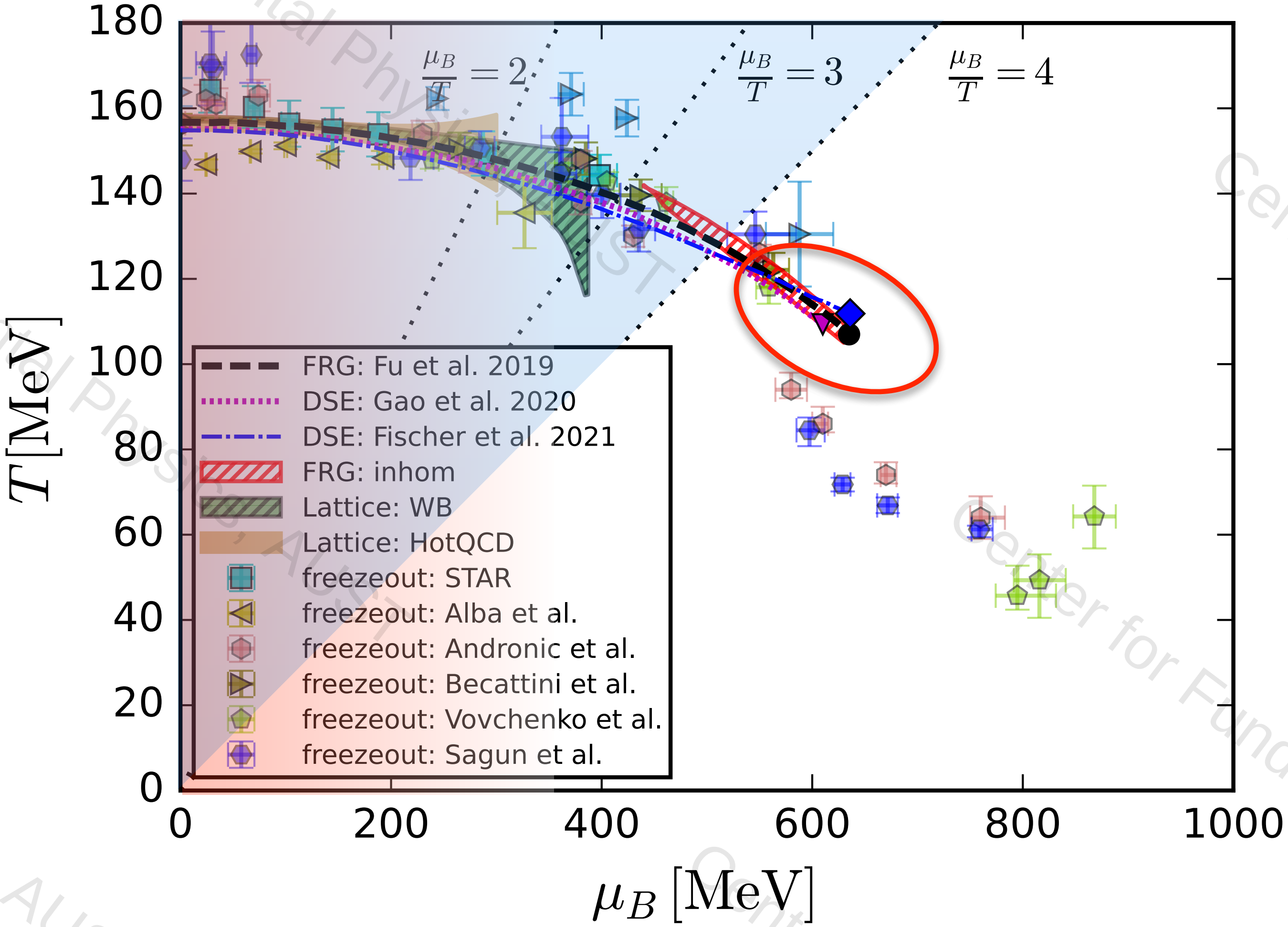
$$\kappa = 0.0149 \pm 0.0021$$

Lattice: Bellwied *et al.* (WB), *PLB* 751 (2015) 559

$$\kappa = 0.015 \pm 0.004$$

Lattice: Bazavov *et al.* (HotQCD), *PLB* 795 (2019) 15

CEP from first-principles functional QCD



Passing through strict benchmark tests in comparison to lattice QCD at vanishing and small μ_B.

Regime of quantitative reliability of functional QCD with $\mu_B/T \lesssim 4$.

Estimates of the location of CEP from first-principles functional QCD:

fRG:

● $(T,\mu_B)_{\mathrm{CEP}} = (107, 635)\mathbf{MeV}$

fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032

DSE:

▼ $(T,\mu_B)_{\mathrm{CEP}} = (109, 610)\mathbf{MeV}$

DSE (fRG): Gao, Pawlowski, *PLB* 820 (2021) 136584

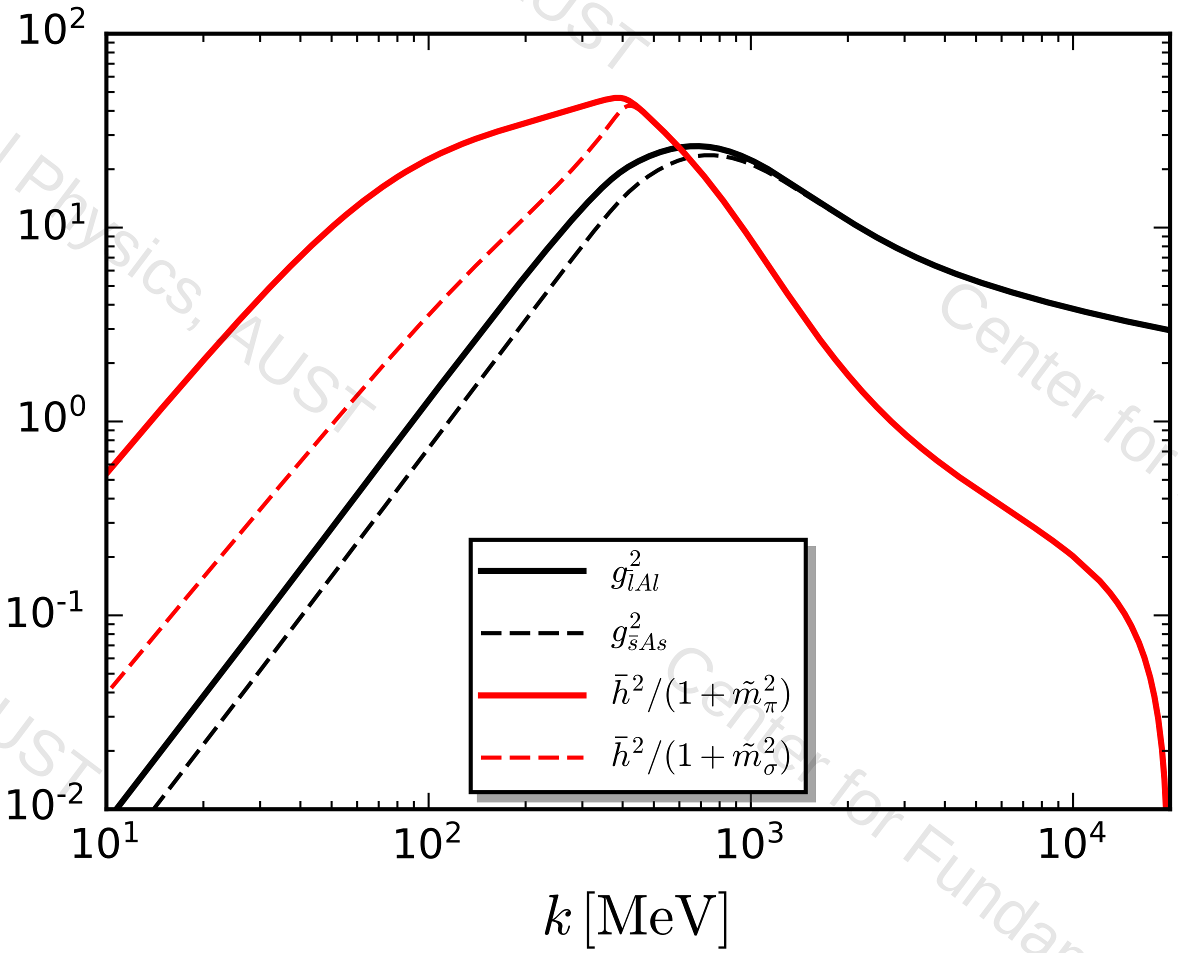
◆ $(T,\mu_B)_{\mathrm{CEP}} = (112, 636)\mathbf{MeV}$

DSE: Gunkel, Fischer, *PRD* 104 (2021) 5, 054022

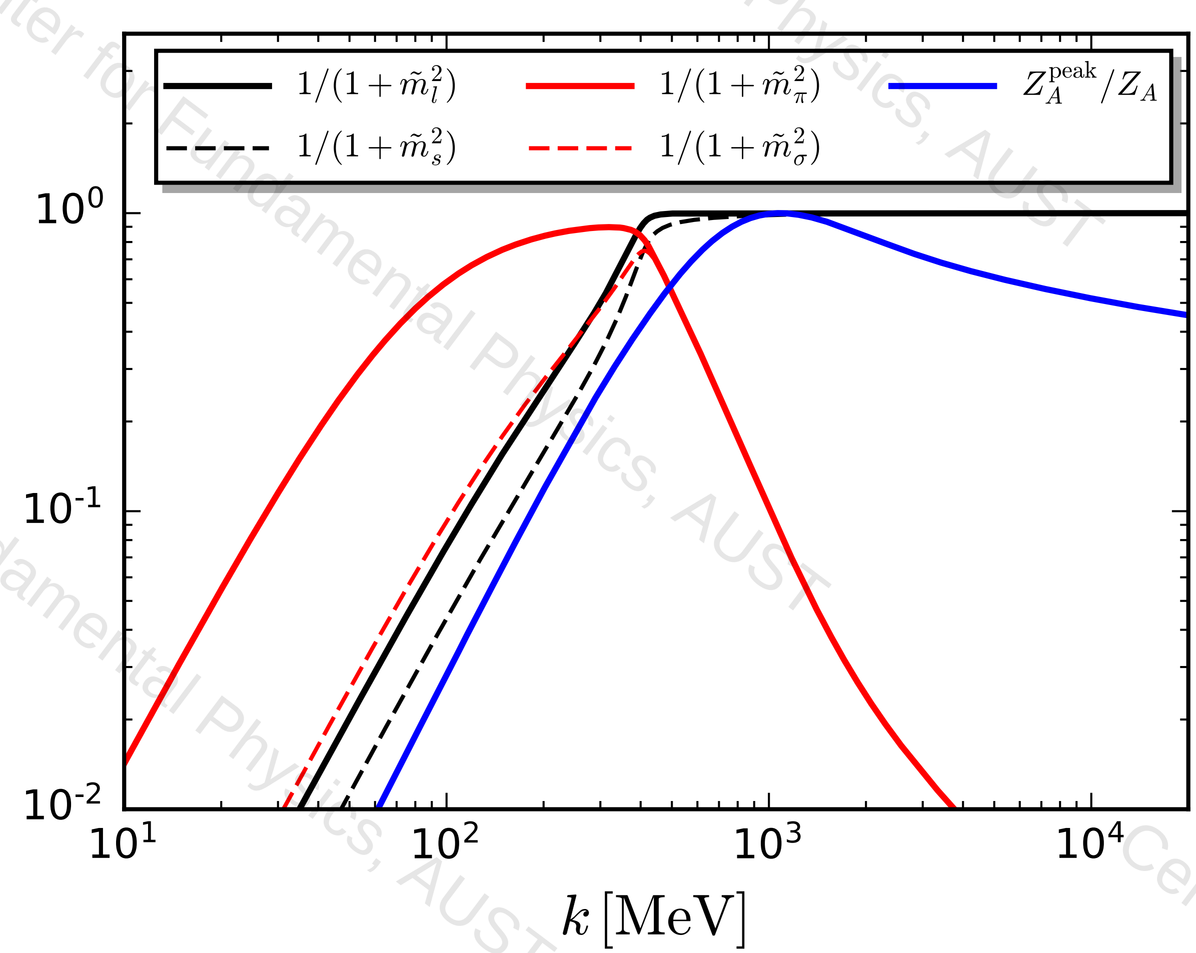
- No CEP observed in $\mu_B/T \lesssim 2 \sim 3$ from lattice QCD. Karsch, *PoS* CORFU2018 (2019)163
- Recent studies of QCD phase structure from both fRG and DSE have shown convergent estimate for the location of CEP: $600\ \mathrm{MeV} \lesssim \mu_{B_{\mathrm{CEP}}} \lesssim 650\ \mathrm{MeV}$.

Natural emergence of LEFTs from QCD

- **Exchange couplings**



- **Propagator gapping**

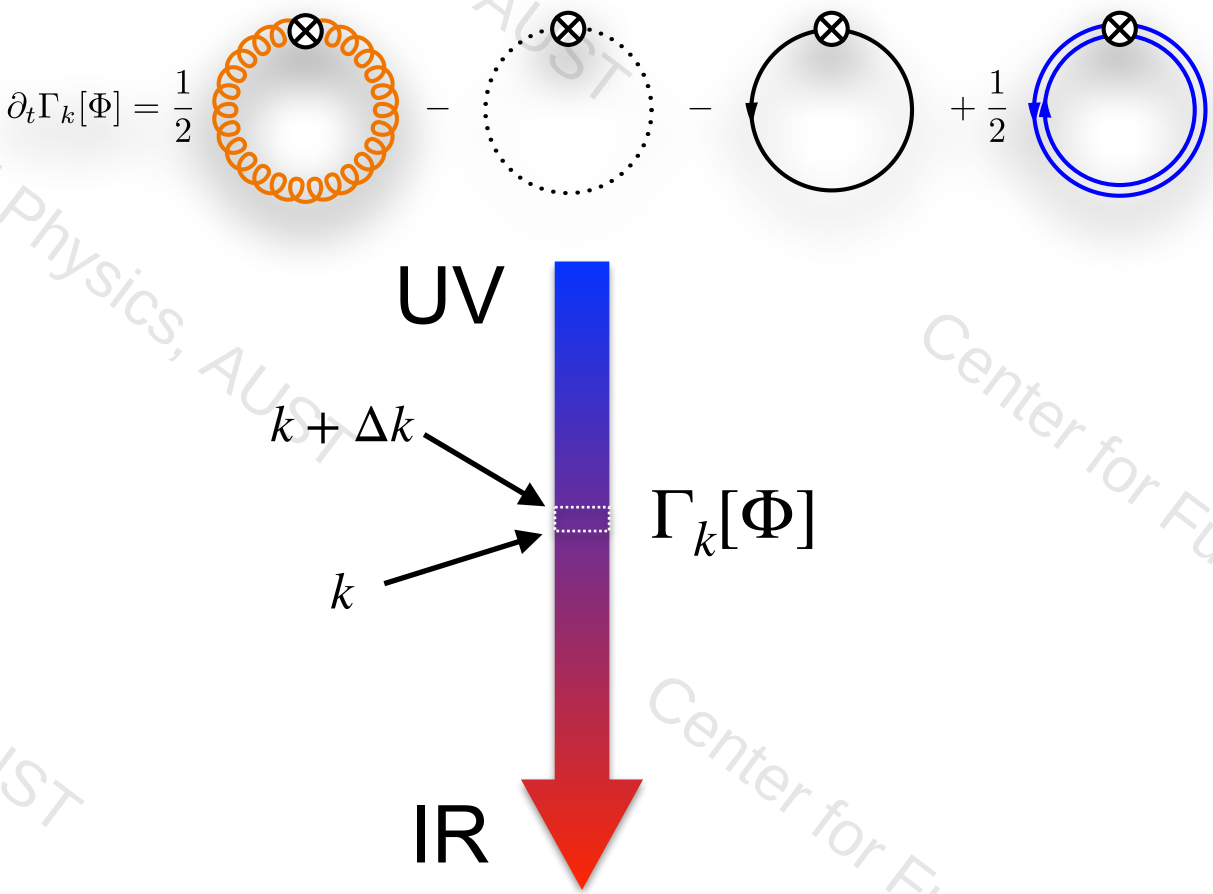


- Composite (mesonic) degrees of freedom take over active dynamics from partonic ones when the RG scale is lowered down $k \lesssim 600 \sim 800$ MeV.
- LEFTs emerge naturally from fundamental theory in the regime of low energy, in agreement with the viewpoint of RG.

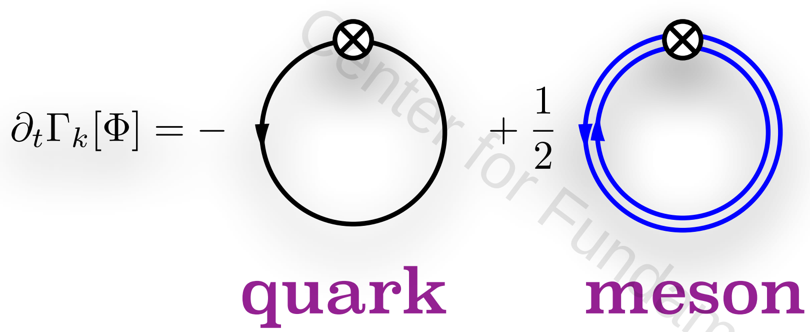
WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

QCD-assisted LEFT

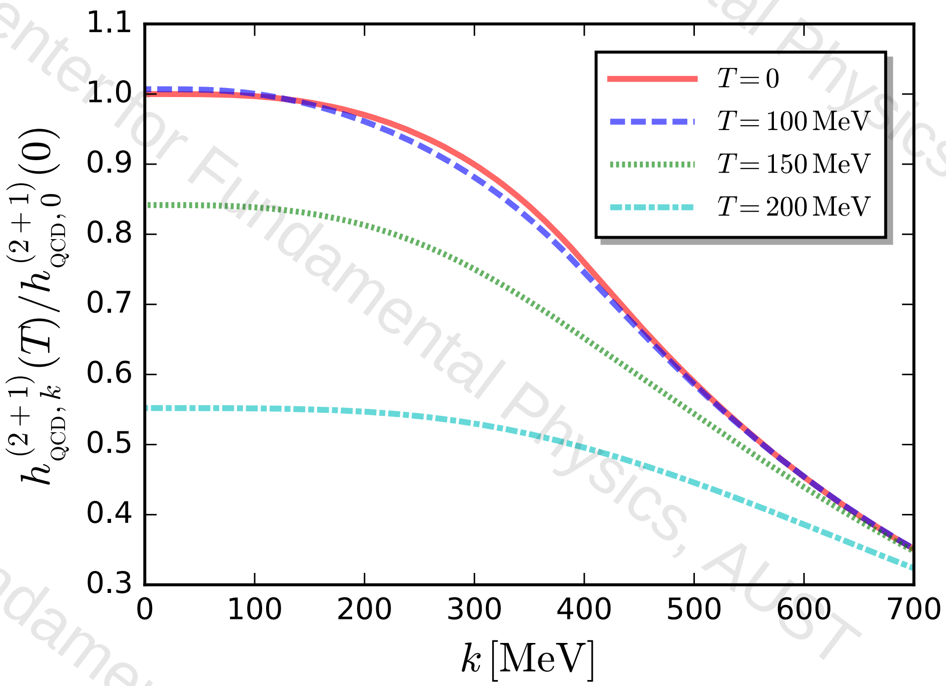
QCD flow equation:



LEFT flow equation:

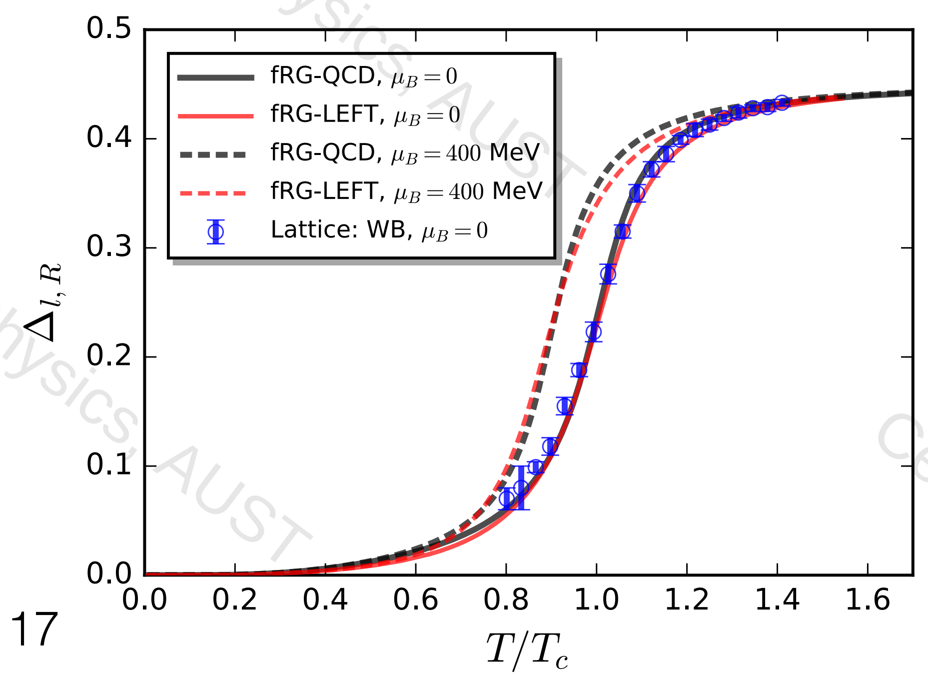


- **Yukawa couplings obtained in QCD inputted in QCD-assisted LEFT**



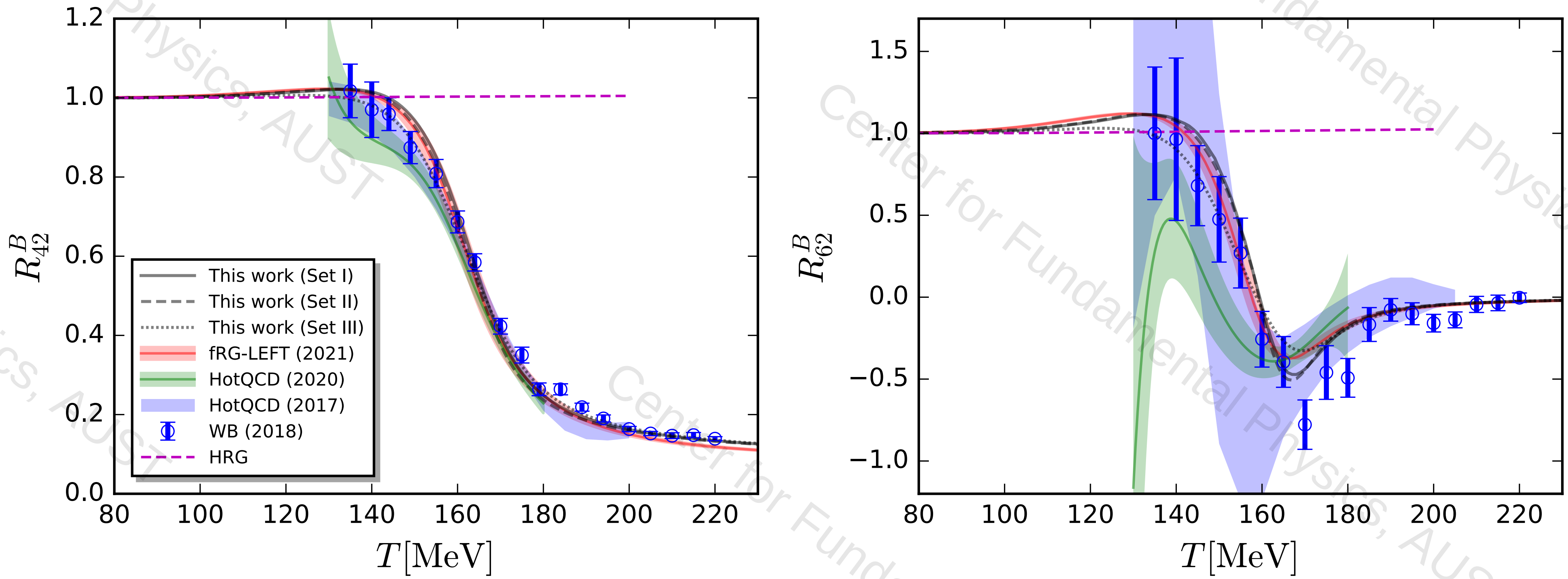
WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

- **Chiral condensates in QCD and QCD-assisted LEFT in agreement**



WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

Baryon number fluctuations



$\mu_B = 0$

fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508;
WF, Luo, Pawlowski, Rennecke, Wen, Yin, *PRD* 104 (2021) 094047

HotQCD: A. Bazavov *et al.*, arXiv: *PRD* 95 (2017), 054504; *PRD* 101 (2020), 074502

WB: S. Borsanyi *et al.*, arXiv: *JHEP* 10 (2018) 205

baryon number fluctuations

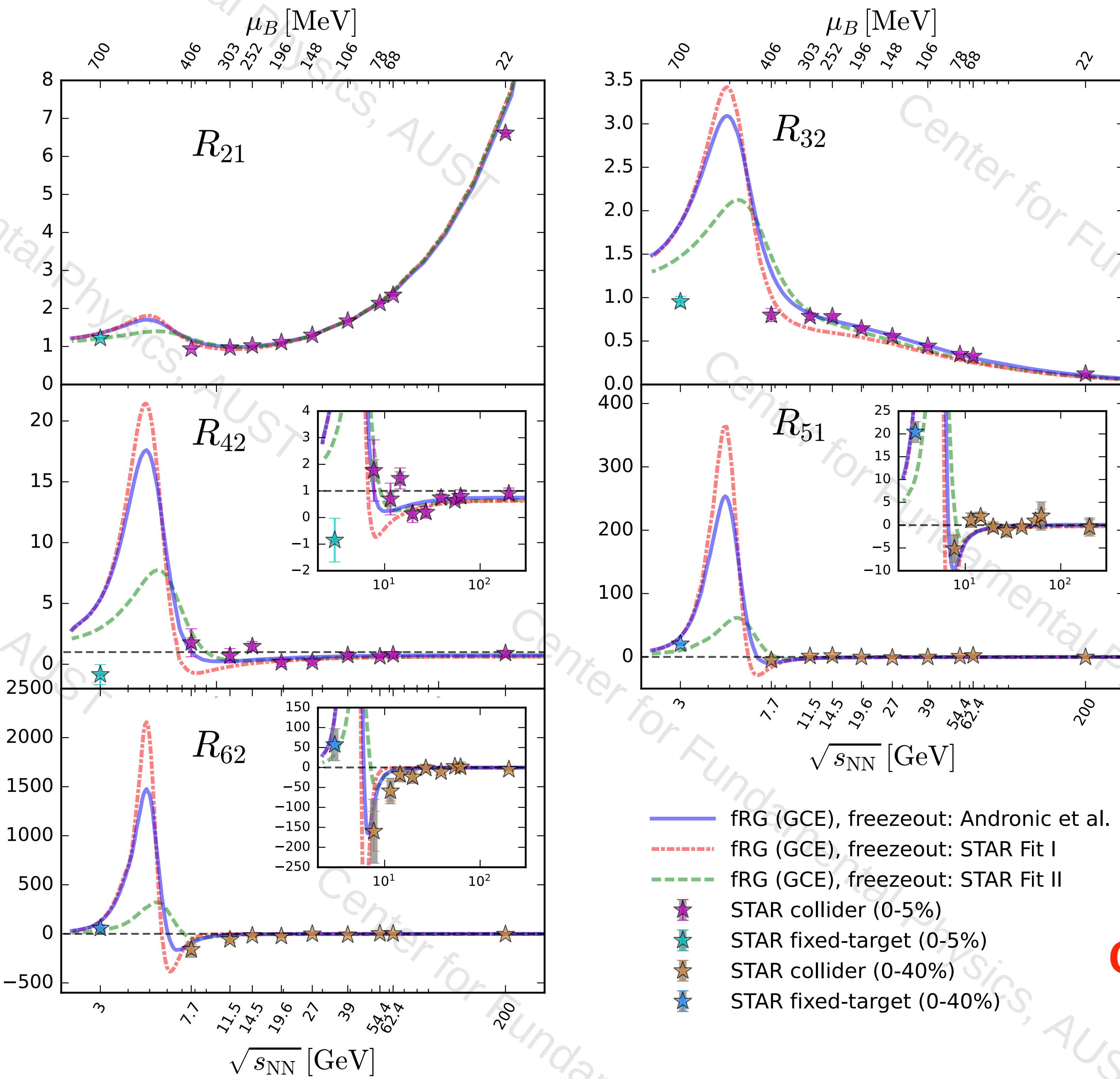
$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4} \qquad R_{nm}^B = \frac{\chi_n^B}{\chi_m^B}$$

relation to the cumulants

$$\frac{M}{VT^3} = \chi_1^B\,,\ \frac{\sigma^2}{VT^3} = \chi_2^B\,,\ S = \frac{\chi_3^B}{\chi_2^B\sigma}\,,\ \kappa = \frac{\chi_4^B}{\chi_2^B\sigma^2}\,,$$

- **In comparison to lattice results and our former results, the improved results of baryon number fluctuations at vanishing chemical potential in the QCD-assisted LEFT are convergent and consistent.**

Grand canonical fluctuations at the freeze-out



STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301;
Abdallah *et al.* (STAR), *PRL* 128 (2022) 202303;
Aboona *et al.* (STAR), *PRL* 130 (2023) 082301

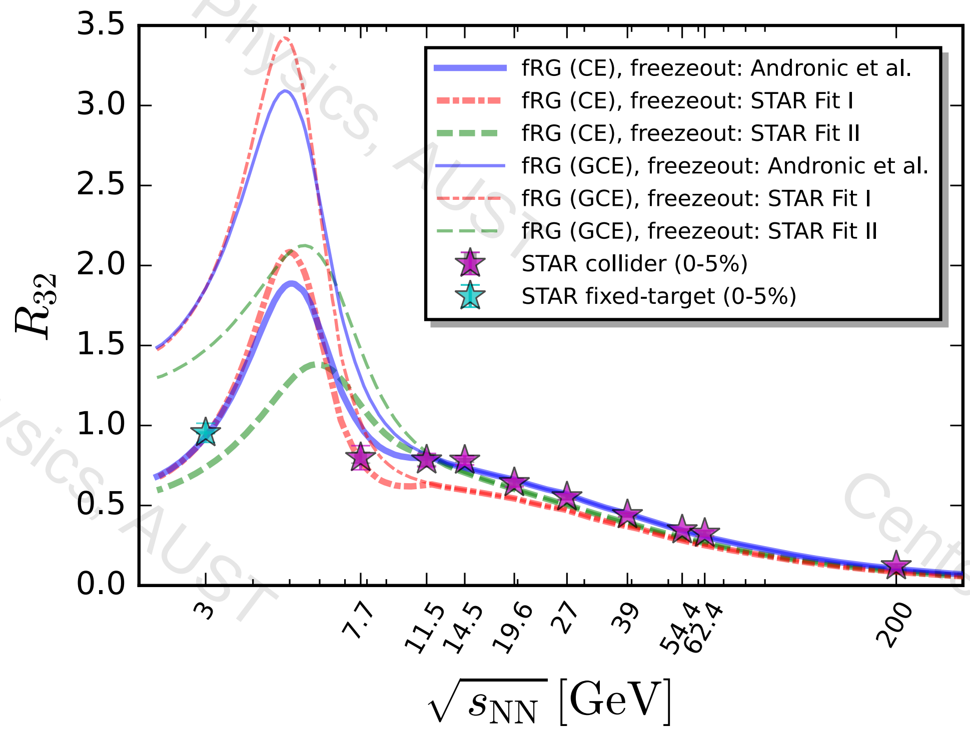
fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

- **Results in fRG are obtained in the QCD-assisted LEFT with a CEP at $(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}}) = (98{,}643)$ MeV.**
- **Peak structure is found in 3 GeV $\lesssim \sqrt{s_{\mathrm{NN}}} \lesssim$ 7.7 GeV.**
- **Agreement between the theory and experiment is worsening with $\sqrt{s_{\mathrm{NN}}} \lesssim$ 11.5 GeV.**
- **Effects of global baryon number conservation in the regime of low collision energy should be taken into account.**

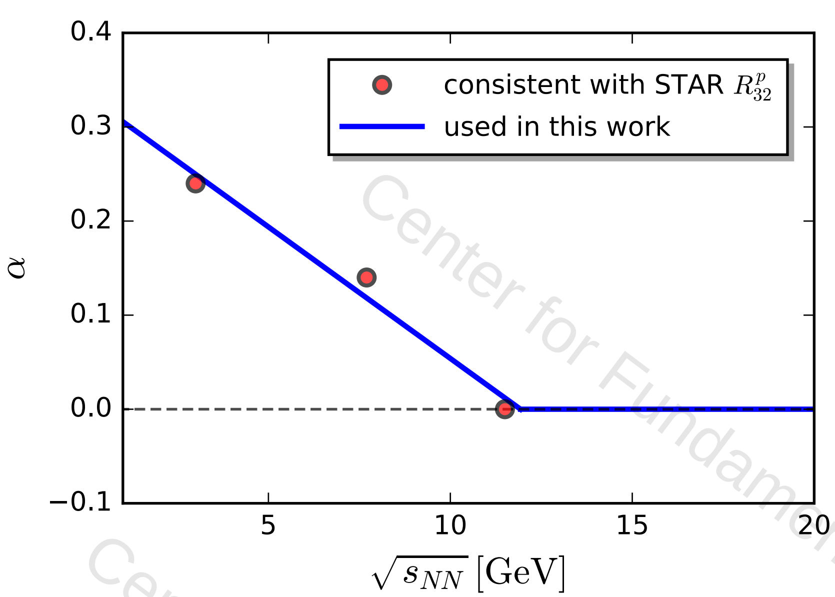
Caveat:

Fluctuations of baryon number in theory are compared with those of proton number in experiments.

Canonical corrections with SAM



- **Experimental data R_{32} is used to constrain the parameter α in the range $\sqrt{s_{\mathrm{NN}}} \lesssim 11.5$ GeV.**
- **We choose the simplest linear dependence**



$$\alpha(\bar{s}) = a\left(1 - \sqrt{\bar{s}}\right)\theta(1 - \bar{s})$$

$$a = 0.33\,, \qquad \sqrt{\bar{s}} = \frac{\sqrt{s_{\mathrm{NN}}}}{11.9\,\mathrm{GeV}}$$

SAM:

- We adopt the subensemble acceptance method (SAM) to take into account the effects of global baryon number conservation:

$$\alpha = \frac{V_1}{V}$$

V_1: the subensemble volume measured in the acceptance window, V: the volume of the whole system.

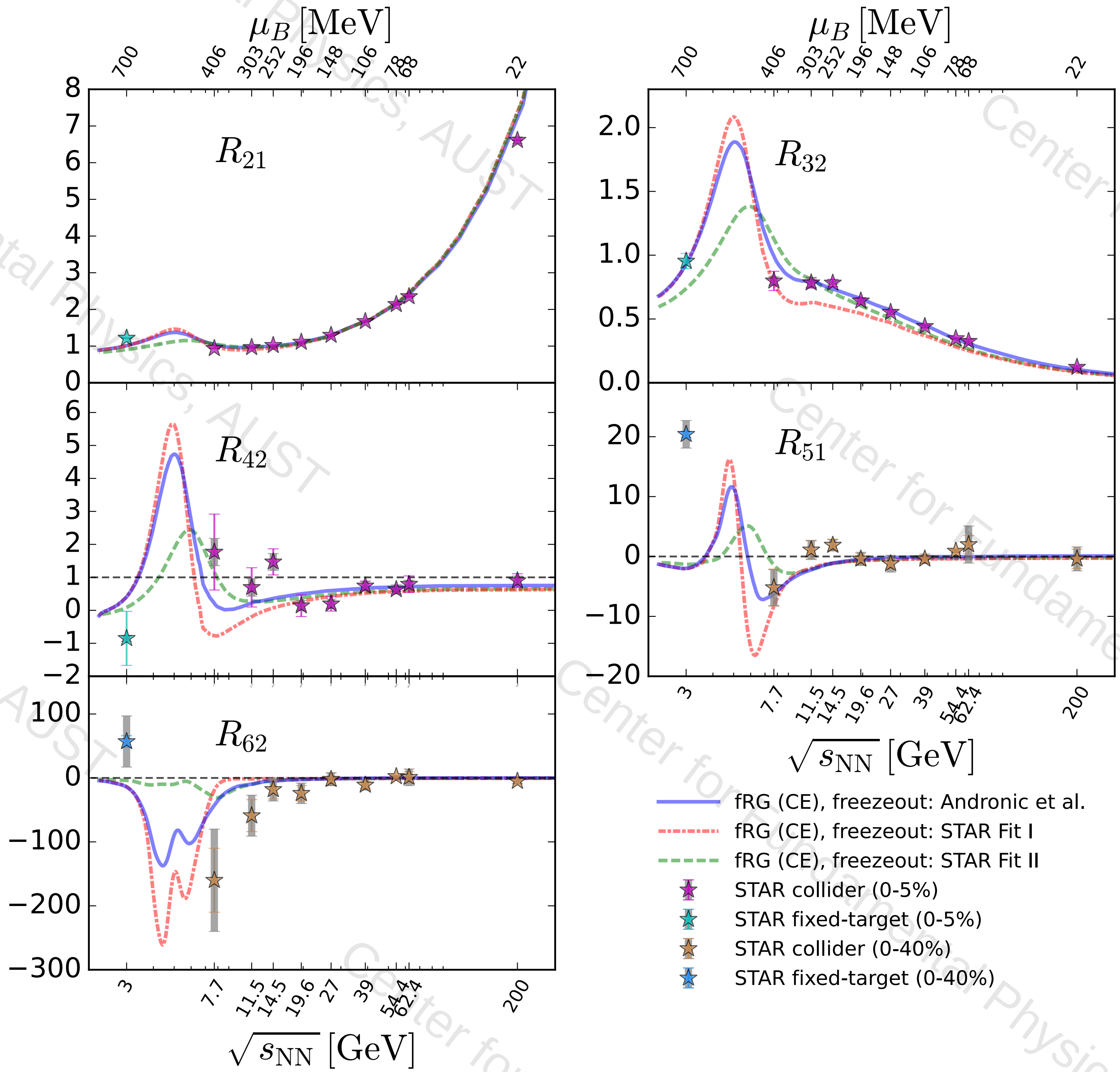
- fluctuations with canonical corrections are related to grand canonical fluctuations as follows:

$$\bar{R}^B_{21} = \beta R^B_{21}, \qquad \bar{R}^B_{32} = (1 - 2\alpha)R^B_{32},$$

$$\bar{R}^B_{42} = (1 - 3\alpha\beta)R^B_{42} - 3\alpha\beta(R^B_{32})^2$$

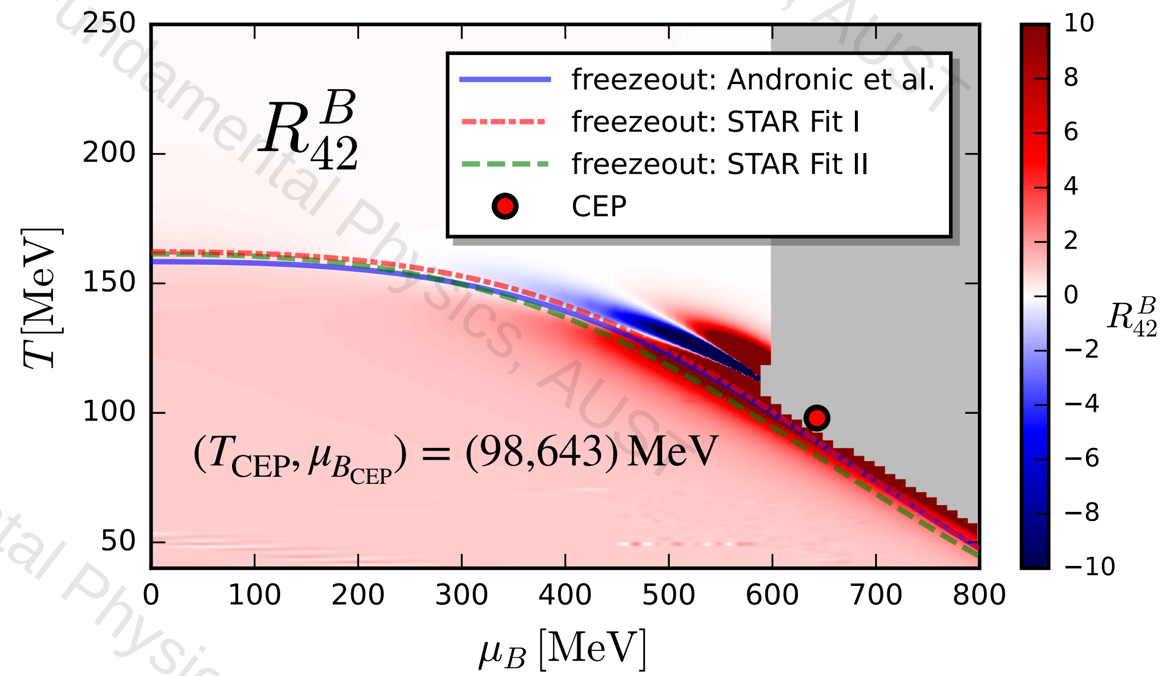
SAM: Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch , *PLB* 811 (2020) 135868

Canonical fluctuations at the freeze-out



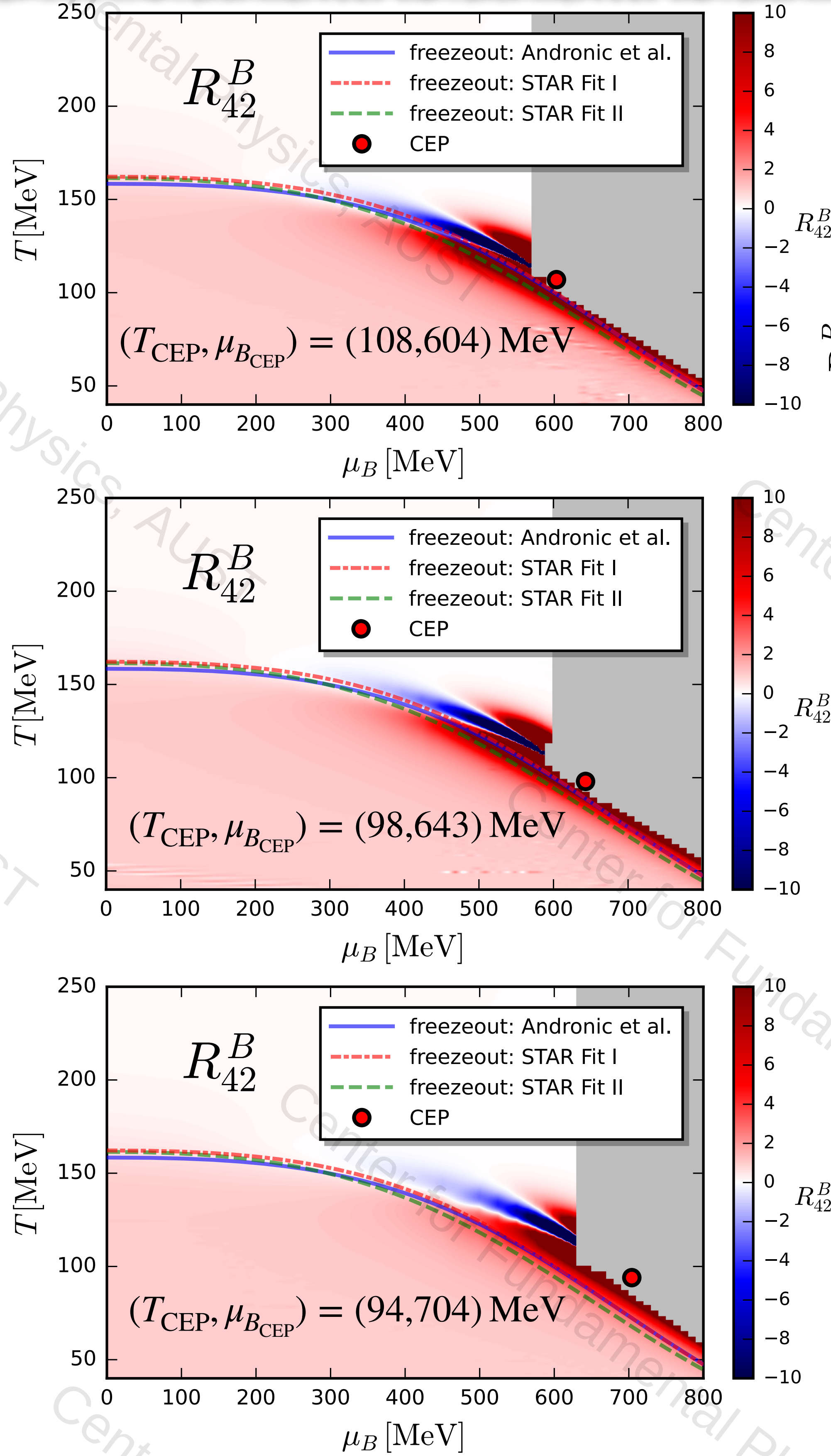
STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301; Abdallah *et al.* (STAR), *PRL* 128 (2022) 202303; Aboona *et al.* (STAR), *PRL* 130 (2023) 082301

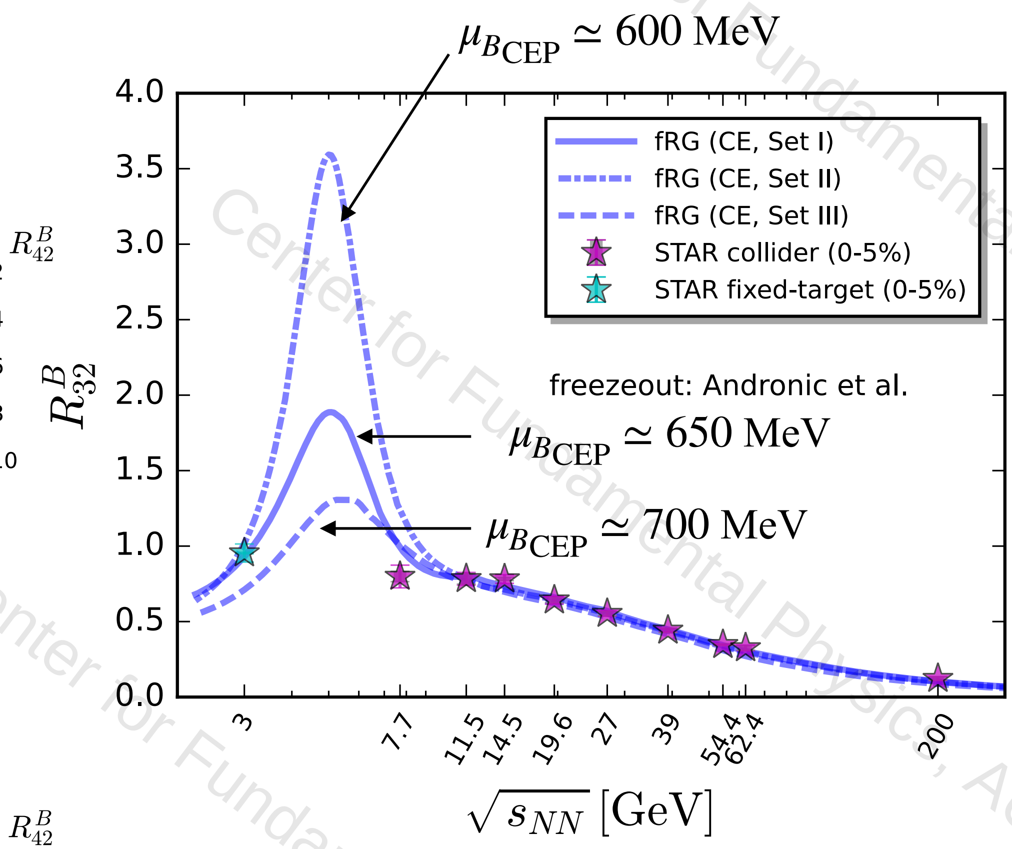
fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508



- **Peak structure is found in 3 GeV $\lesssim \sqrt{s_{NN}} \lesssim$ 7.7 GeV.**
- **Position of peak in R_{42} is $\mu_{B_{\rm peak}}$ = 536, 541 and 486 MeV for the three freeze-out curves, significantly smaller than $\mu_{B_{CEP}}$ = 643 MeV.**

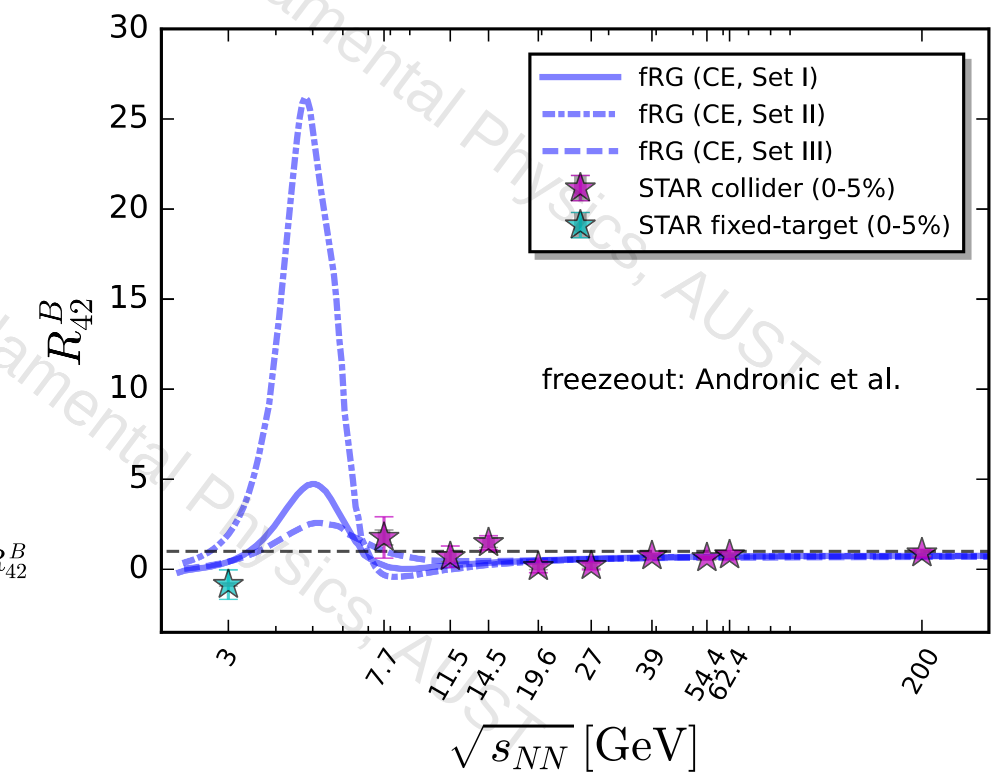
Dependence on the location of the CEP





STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301

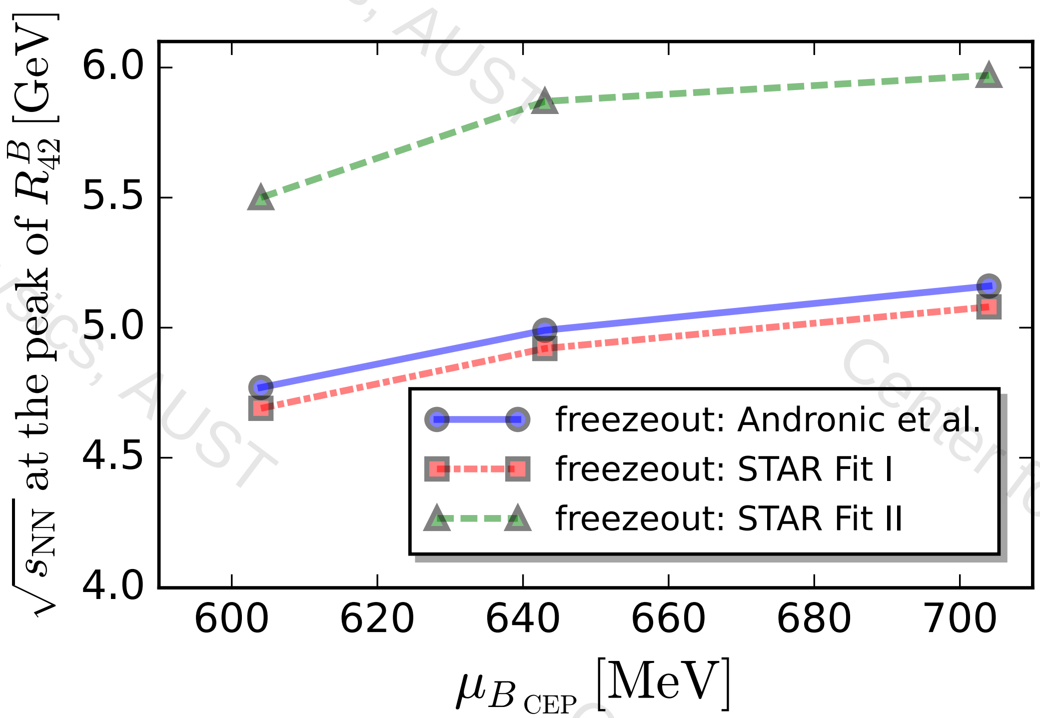
fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508



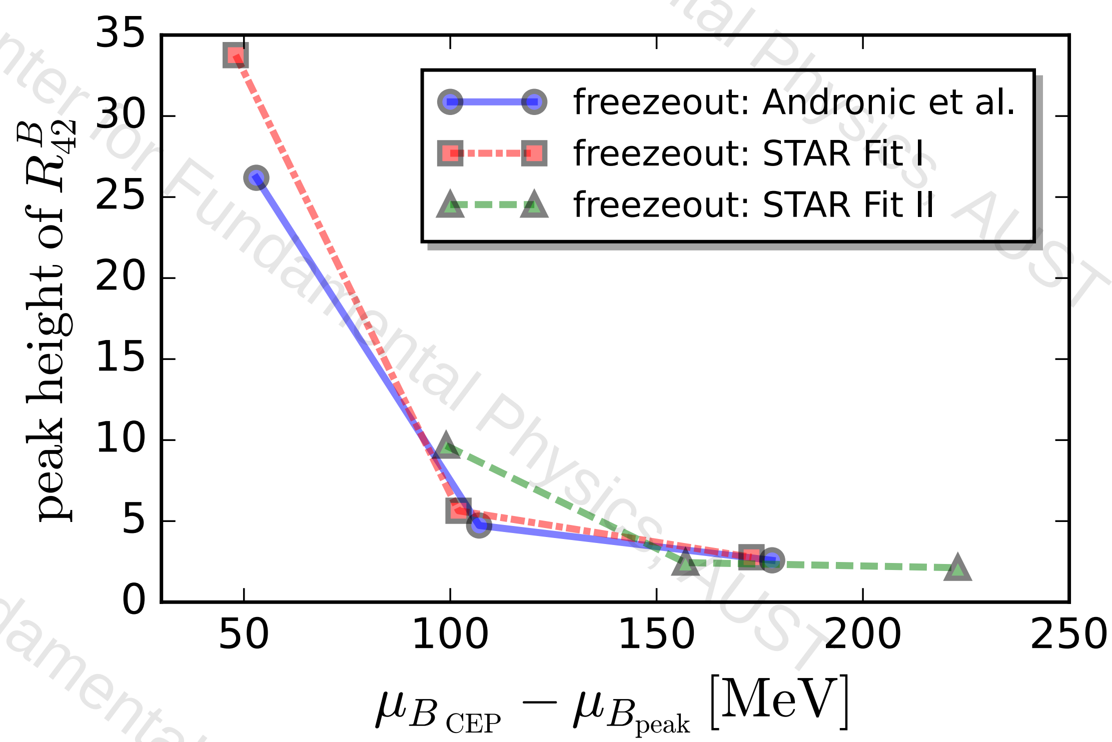
- **Position of the peak is insensitive to the location of CEP.**
- **Height of peak decreases as CEP moves towards larger μ_B.**

Ripples of the QCD critical point

Postion of peak:



Height of peak:



fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

- **Note that the ripples of CEP are far away from the critical region characterized by the universal scaling properties, e.g., the critical exponents.**
- **But, the information of CEP, such as its location and properties, etc., is still encoded in the ripples.**

Magnetic equation of state

- The magnetic equation of state (EoS) is obtained via the chiral condensate:

$$\Delta_q = m_q \frac{\partial \Omega\big(T; m_q(T)\big)}{\partial m_q} = m_q \frac{T}{V} \int_x \big\langle \bar{q}(x)\, q(x) \big\rangle$$

- The chiral properties of the magnetic EoS are encoded in the magnetic susceptibility:

$$\chi_M = -\frac{\partial \bar{\Delta}_l}{\partial m_l}, \qquad \text{with} \qquad \bar{\Delta}_l = \frac{\Delta_l}{m_l}$$

- In the critical region, the magnetic EoS can be expressed as a universal scaling function $f_G(z)$ through

$$\bar{\Delta}_l = m_l^{1/\delta} f_G(z)$$

with

$$z = t\, m_l^{-1/\beta\delta}, \qquad \text{and} \qquad t = (T - T_c)/T_c$$

z is the scaling variable and t is the reduced temperature.

- The pseudo-critical temperature T_{pc}, which is defined through the peak location of χ_M, is readily obtained from the scaling function as

$$T_{pc}(m_\pi) \approx T_c + c\, m_\pi^p, \qquad \text{with} \qquad p = 2/(\beta\delta)$$

Critical exponent in fRG for 3d-O(4):

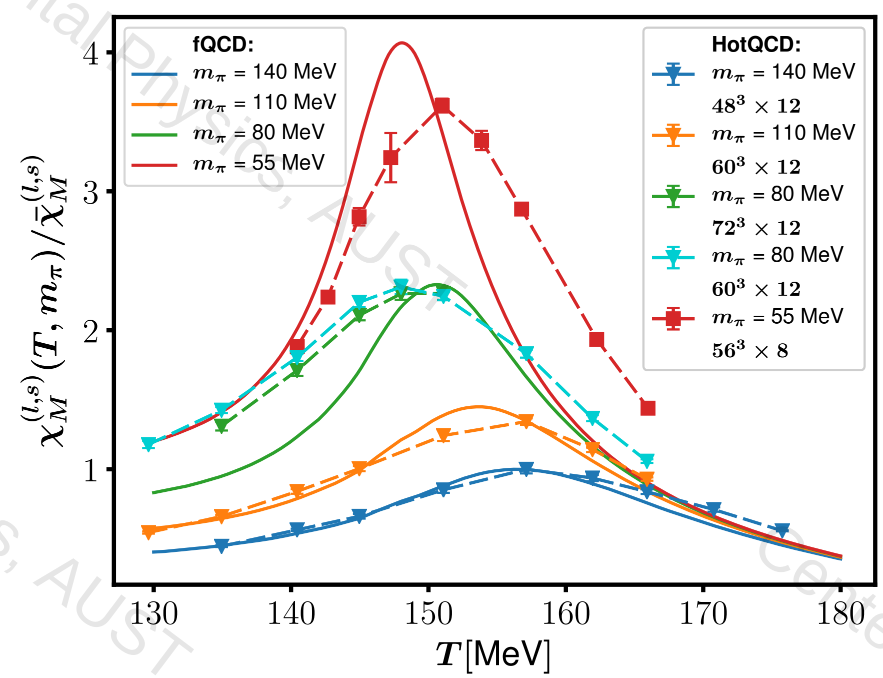
$$\beta = 0.405, \quad \delta = 4.784, \quad \theta_H = 0.272,$$

obtained from the fixed-point equation for the Wilson-Fisher fixed point, which leads us $p_{\mathrm{fRG}} = 1.03$

Critical exponent in mean field:

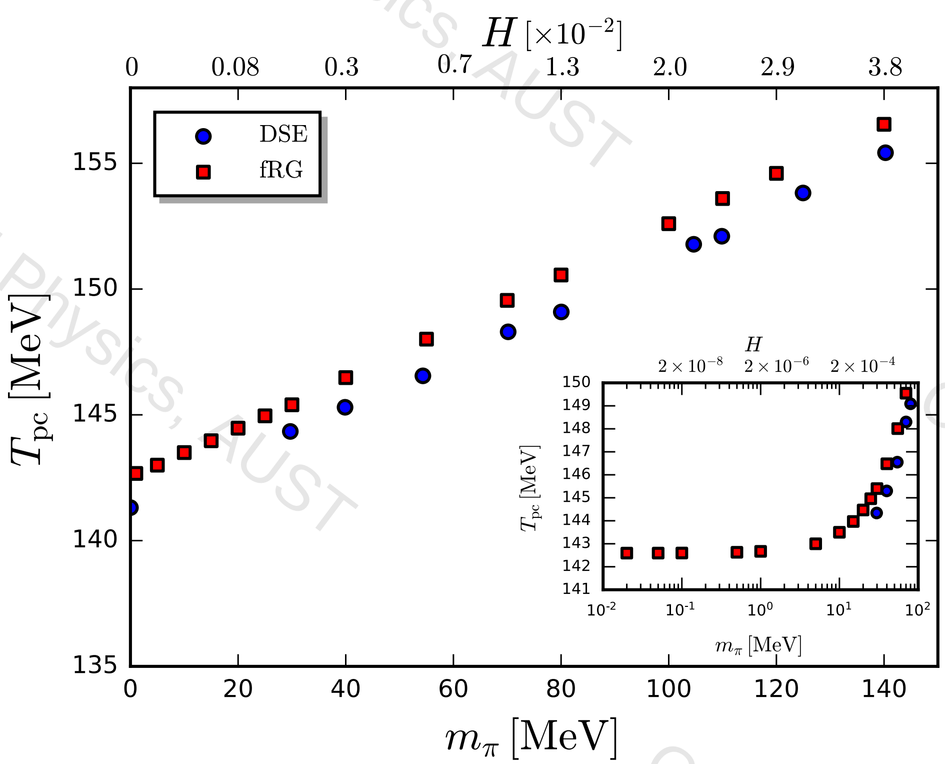
$$\beta_{\mathrm{MF}} = 1/2, \quad \delta_{\mathrm{MF}} = 3,$$

thus, one has $p_{\mathrm{MF}} = 4/3$



Braun, WF, Pawlowski, Rennecke, Rosenblüh, Yin, *PRD* 102 (2020), 056010.

Magnetic equation of state



$$T_{\rm pc}(m_\pi) \approx T_c + c\, m_\pi^p$$

Braun, Chen, WF, Gao, Huang, Ihssen, Pawlowski, Rennecke, Sattler, Tan, Wen, and Yin, arXiv:2310.19853.

Lattice (HotQCD):

$$T_c^{\rm lattice} = 132^{+3}_{-6}\,{\rm MeV},$$

Ding *et al.*, *PRL* 123 (2019) 062002.

fRG:

$$T_c^{\rm fRG} \approx 142\,{\rm MeV}\,, \qquad p_{\rm fRG} = 1.024$$

Braun, WF, Pawlowski, Rennecke, Rosenblüh, Yin, *PRD* 102 (2020) 056010.

DSE:

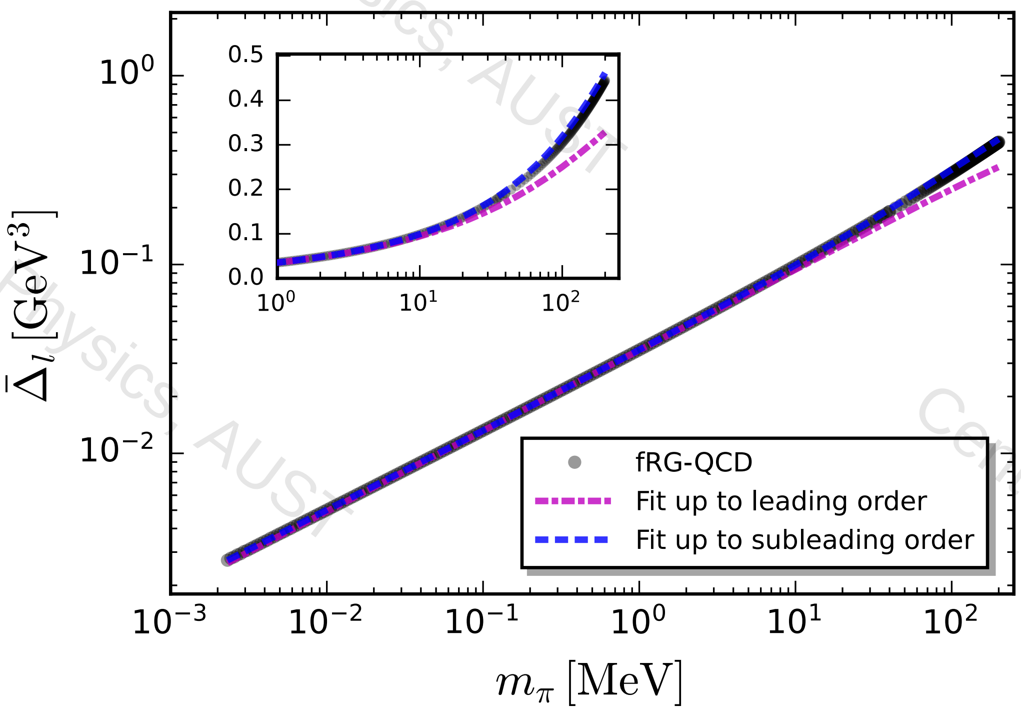
$$T_c^{\rm DSE} \approx 141\,{\rm MeV}\,, \qquad p_{\rm DSE} = 0.9606$$

Gao, Pawlowski, *PRD* 105 (2022) 9, 094020, arXiv: 2112.01395.

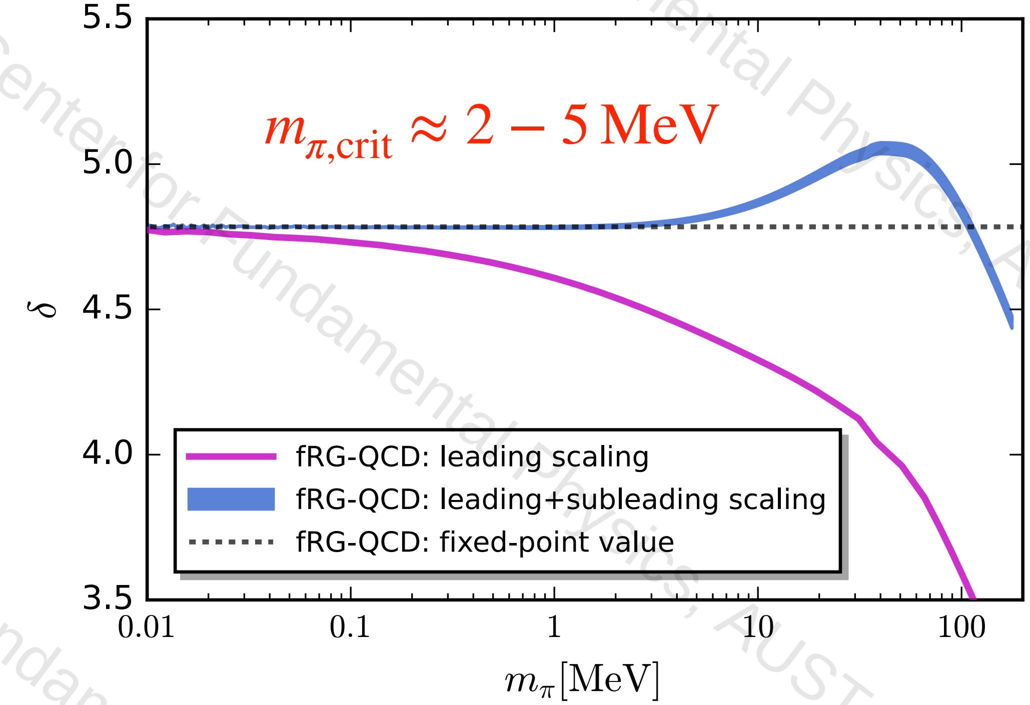
- **The almost linear dependence of the pseudo-critical temperature on the pion mass has nothing to do with the criticality.**
- **So what is the size of the critical region in QCD?**

Critical region in QCD

Scaling in the external field:

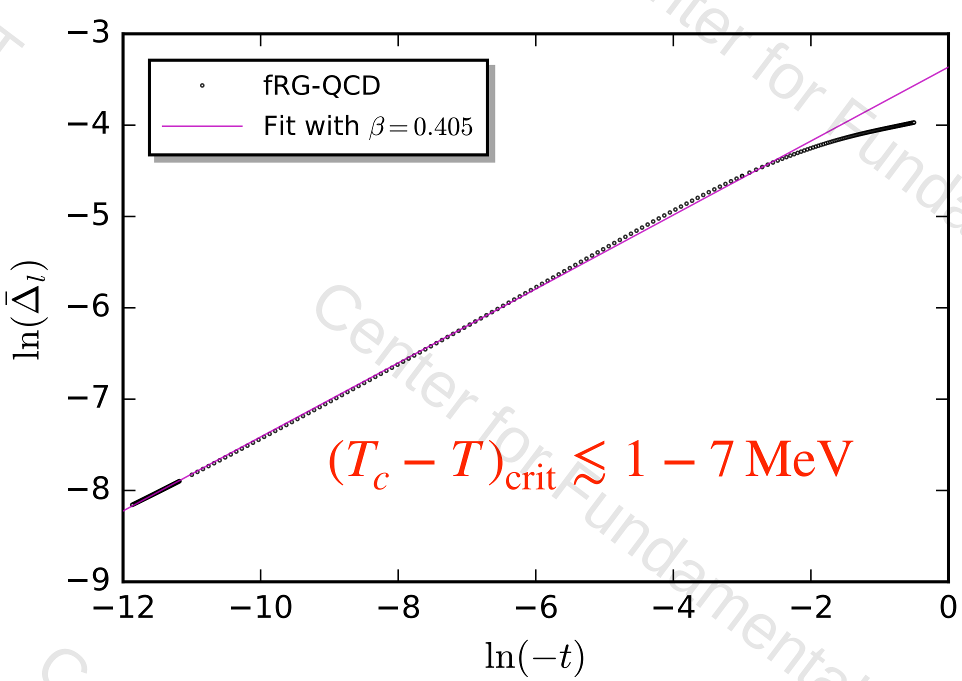


Critical exponent δ:



$$\bar{\Delta}_l^{(\mathrm{crit})}(m_\pi) = B_c\, m_\pi^{2/\delta}\left[1 + a_m m_\pi^{2\theta_H}\right]$$

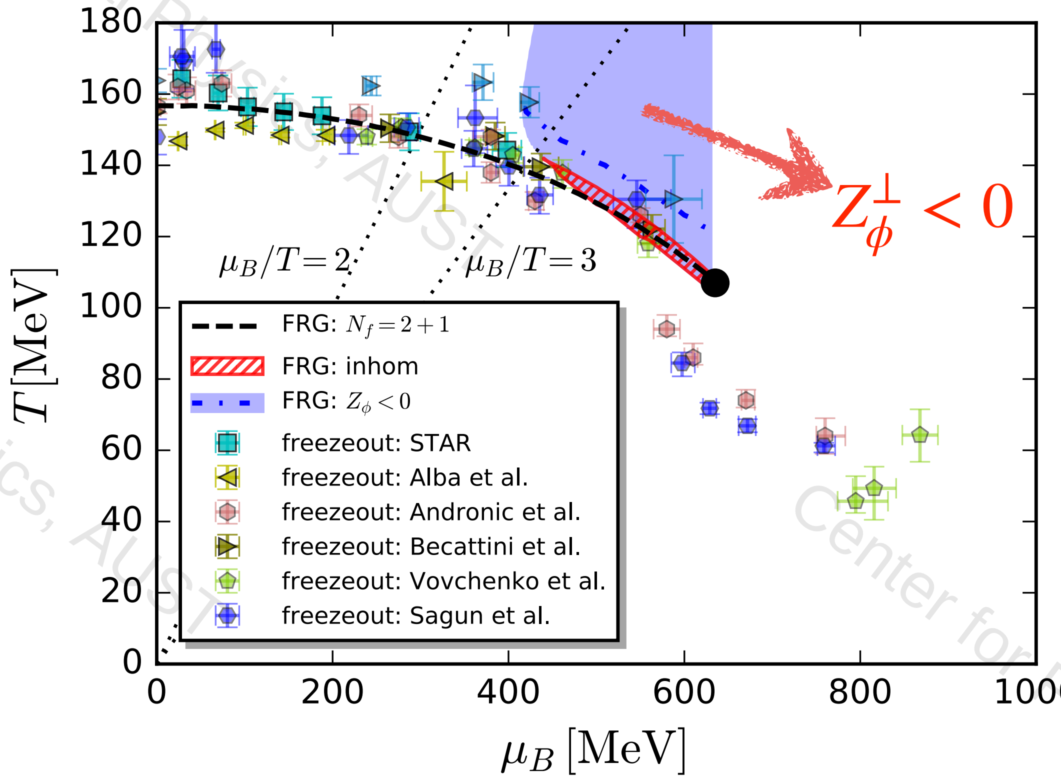
Scaling in the temperature:



- QCD at physical light quark mass is far away from the critical region.
- The scaling behavior is observed for the first time in the calculations of first-principles QCD.

Braun, Chen, WF, Gao, Huang, Ihssen, Pawlowski, Rennecke, Sattler, Tan, Wen, and Yin, arXiv:2310.19853.

Inhomogeneous instabilities in QCD phase diagam



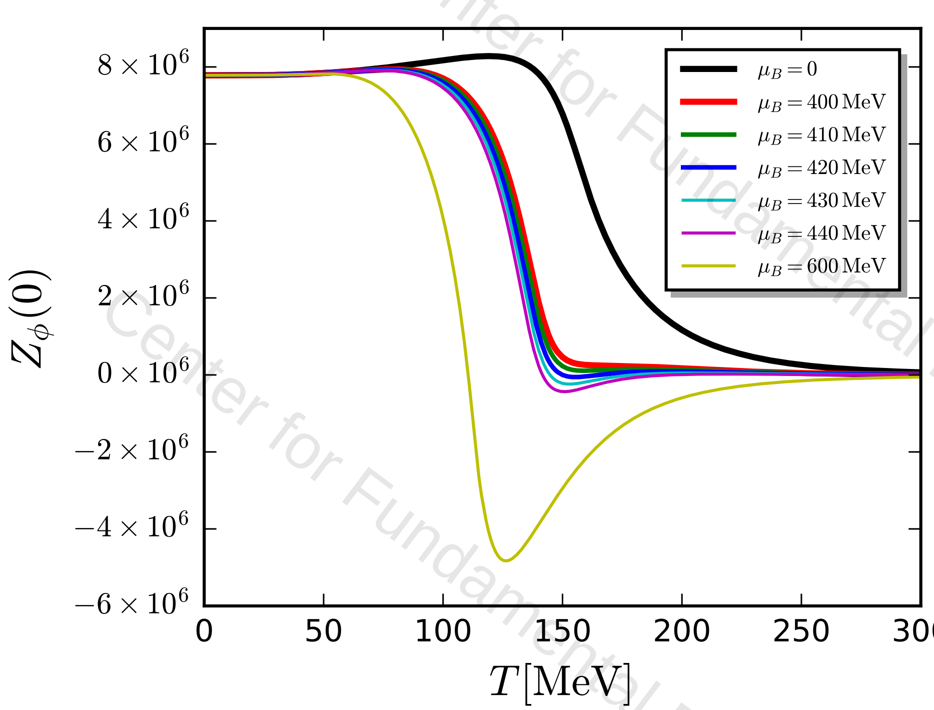
WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032, arXiv:1909.02991

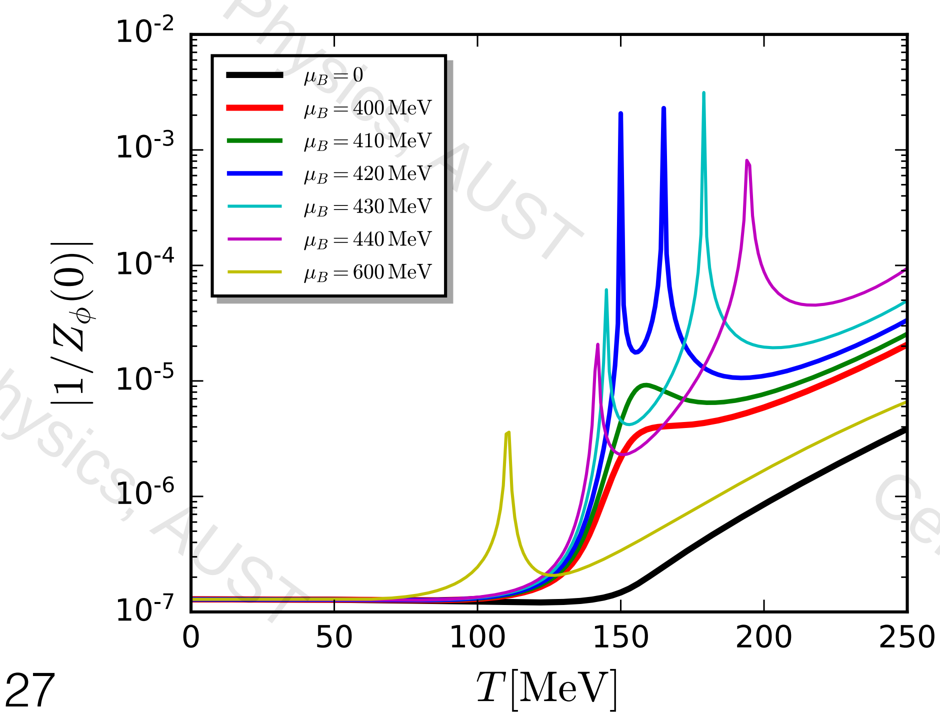
Mesonic two-point correlation function:

$$\Gamma^{(2)}_{\phi\phi}(p) = \left[Z^{\parallel}_{\phi}(p_0,\mathbf{p})\,p_0^2 + Z^{\perp}_{\phi}(p_0,\mathbf{p})\,\mathbf{p}^2\right] + m_\phi^2$$

with

$$\Gamma^{(2)}_{\phi\phi,k} = \frac{\delta^2\Gamma_k[\Phi]}{\delta\phi\delta\phi}\bigg|_{\Phi=\Phi_{\mathrm{EoM}}}$$

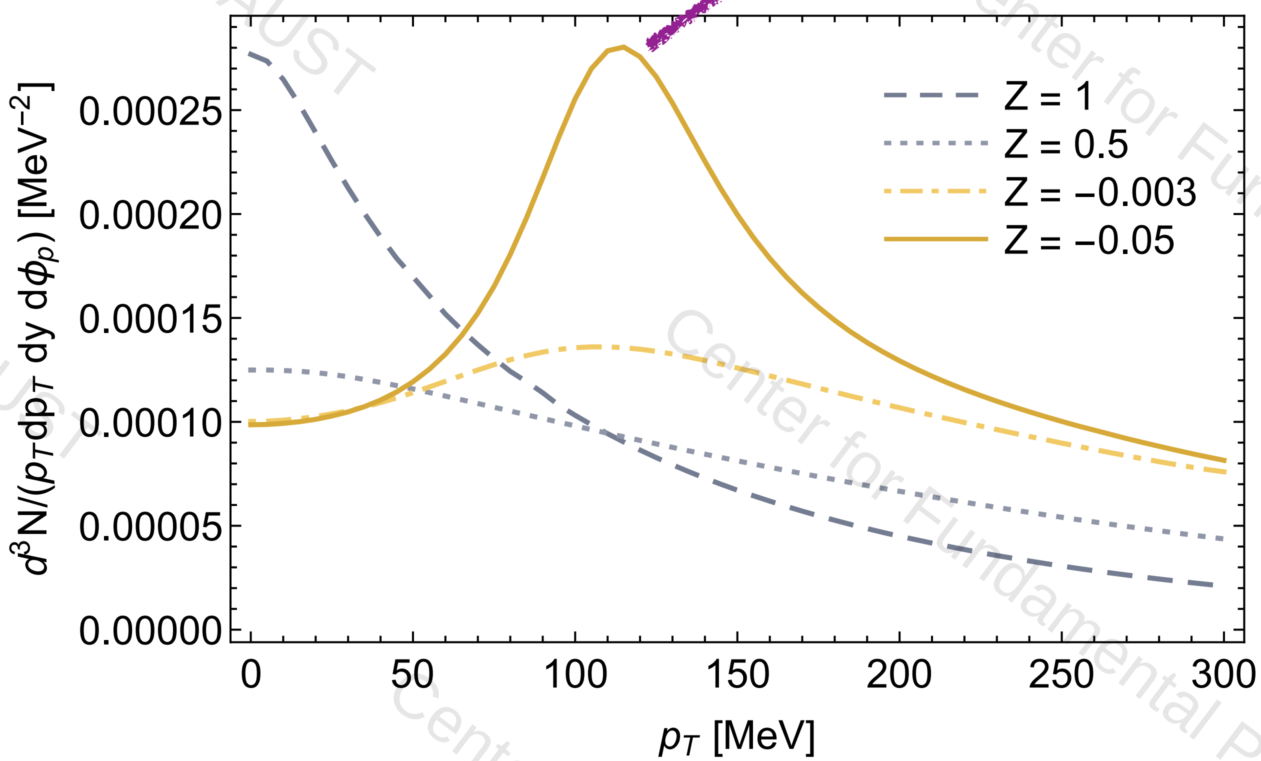




Signature of inhomogeneous instability in heavy-ion collisions—-“moat” spectrum

- **transverse momentum spectrum of one particle:**

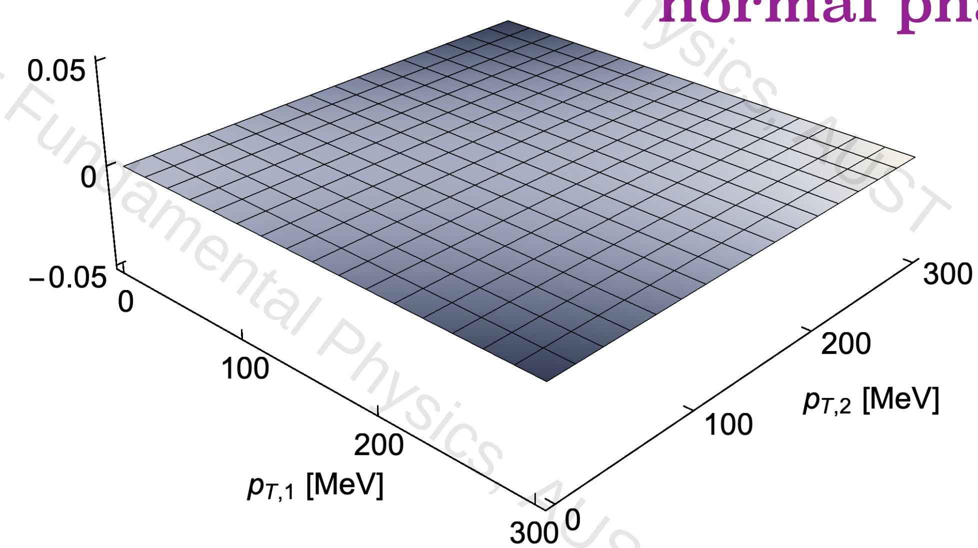
moat behavior



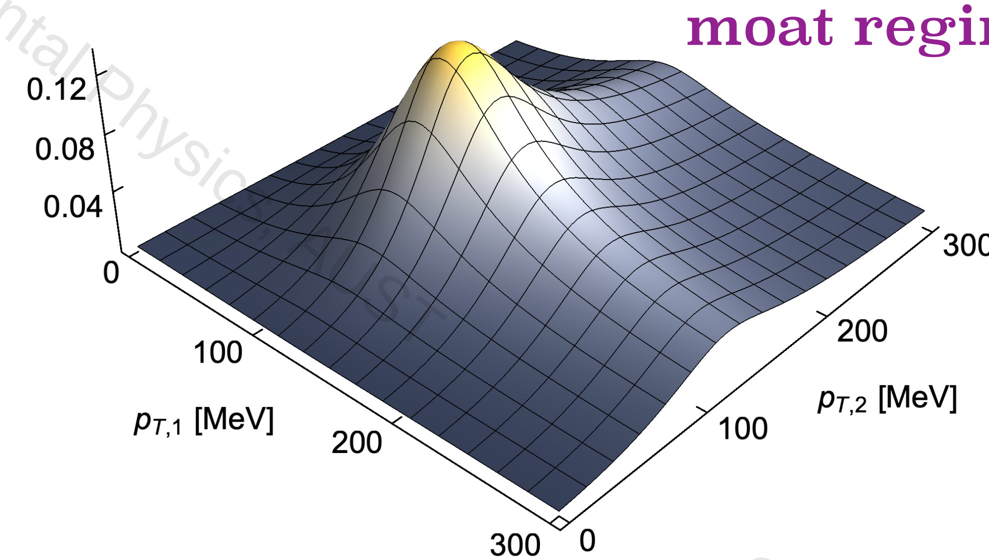
Pisarski, Rennecke, *PRL* 127 (2021) 152302;
Rennecke, Pisarski, arXiv:2110.02625

- **two-particle correlation:**

normal phase



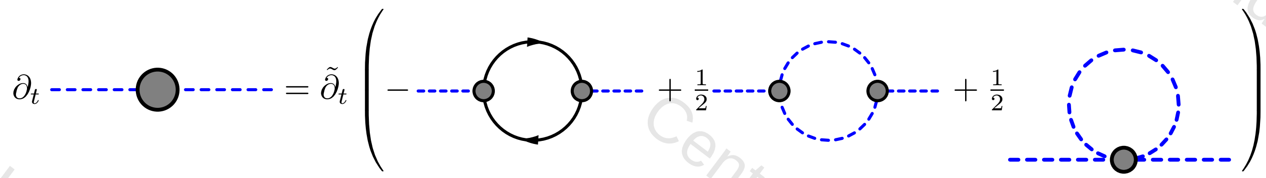
moat regime

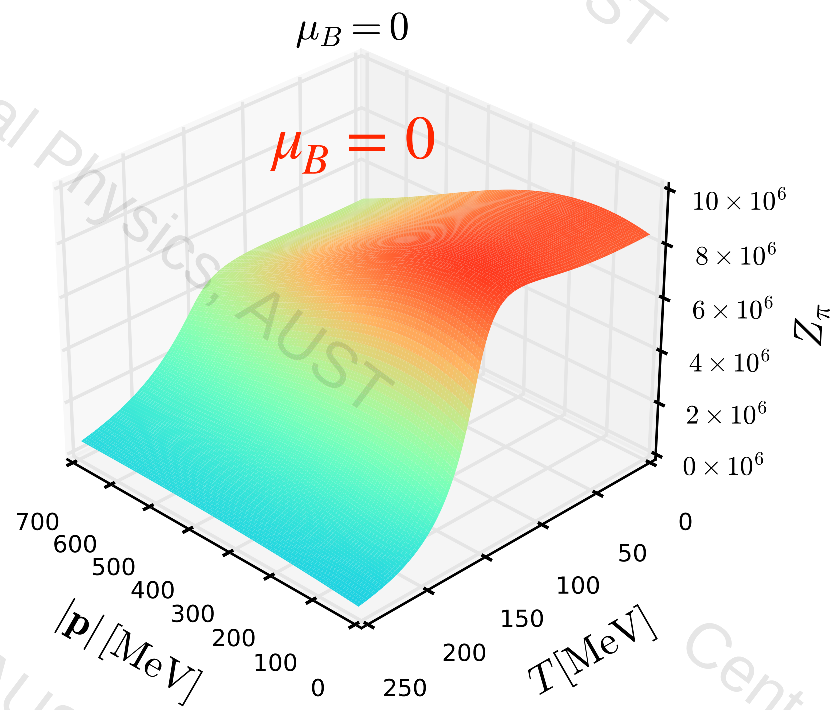


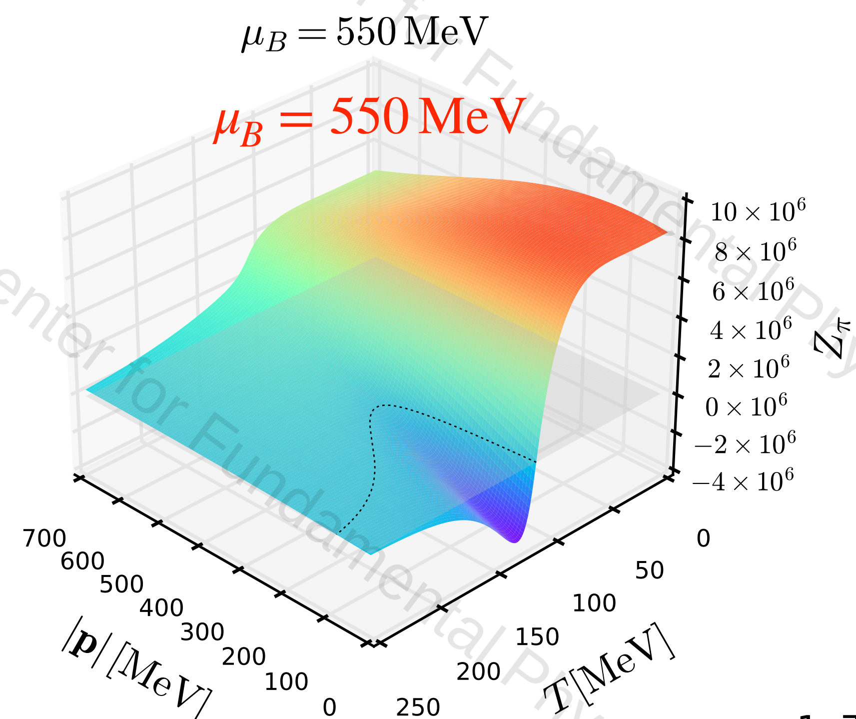
$$\Delta n_{12} = \left\langle \left(\frac{d^3N}{d\mathbf{p}^3}\right)^2 \right\rangle_c \Big/ \left\langle \frac{d^3N}{d\mathbf{p}^3} \right\rangle^2$$

Momentum-dependent mesonic wave function

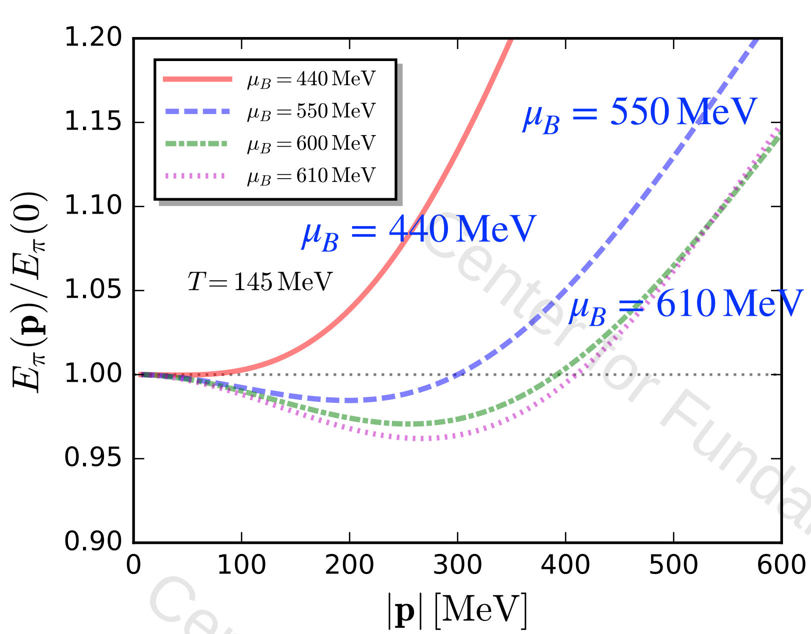
Flow equation for mesonic two-point functions:







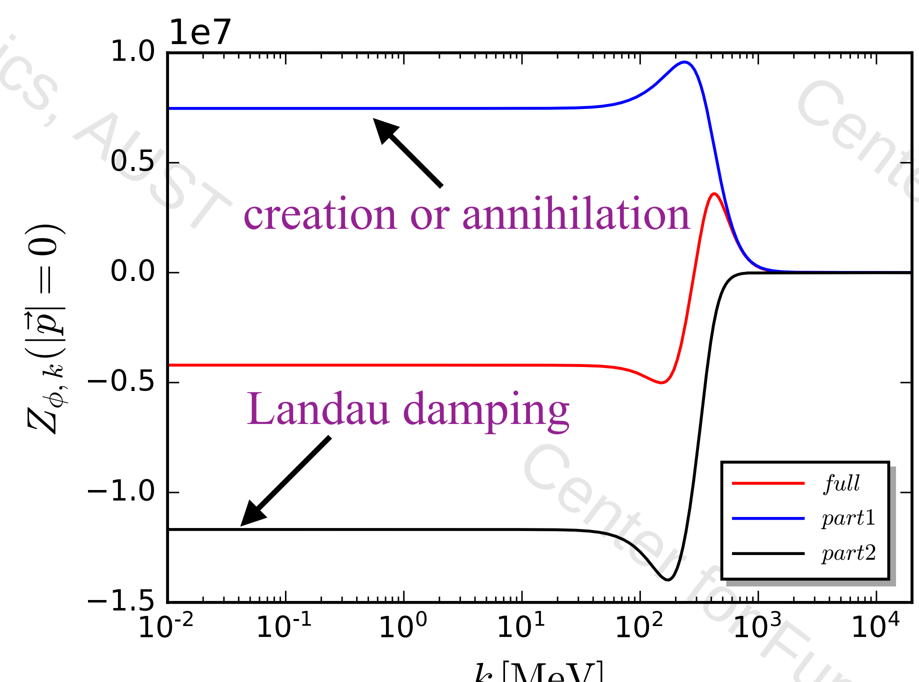
- Inhomogeneous instability is resulted from Landau damping of quarks in thermal bath in the regime of large baryon chemical potential.



Dispersion relation:

$$E_\phi(\mathbf{p}) = \left[Z_\phi^\perp(\mathbf{p})\,\mathbf{p}^2 + m_\phi^2\right]^{1/2}$$

WF, Pawlowski, Pisarski, Rennecke, Wen, Yin, in preparation.

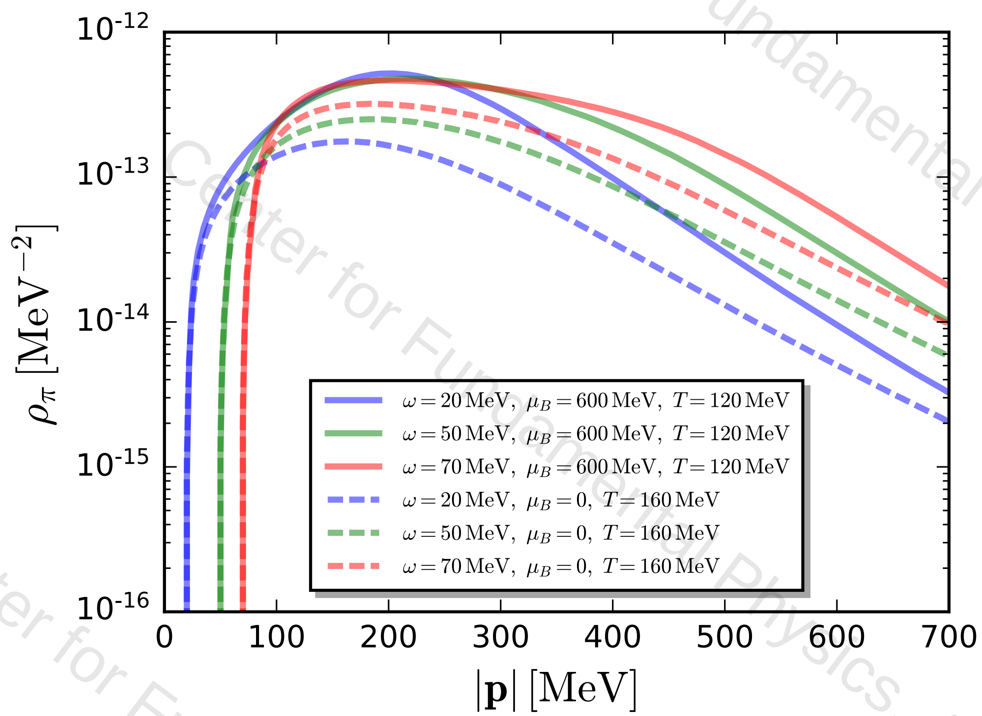


Real-time mesonic two-point functions

Analytic continuation on the flow equation:

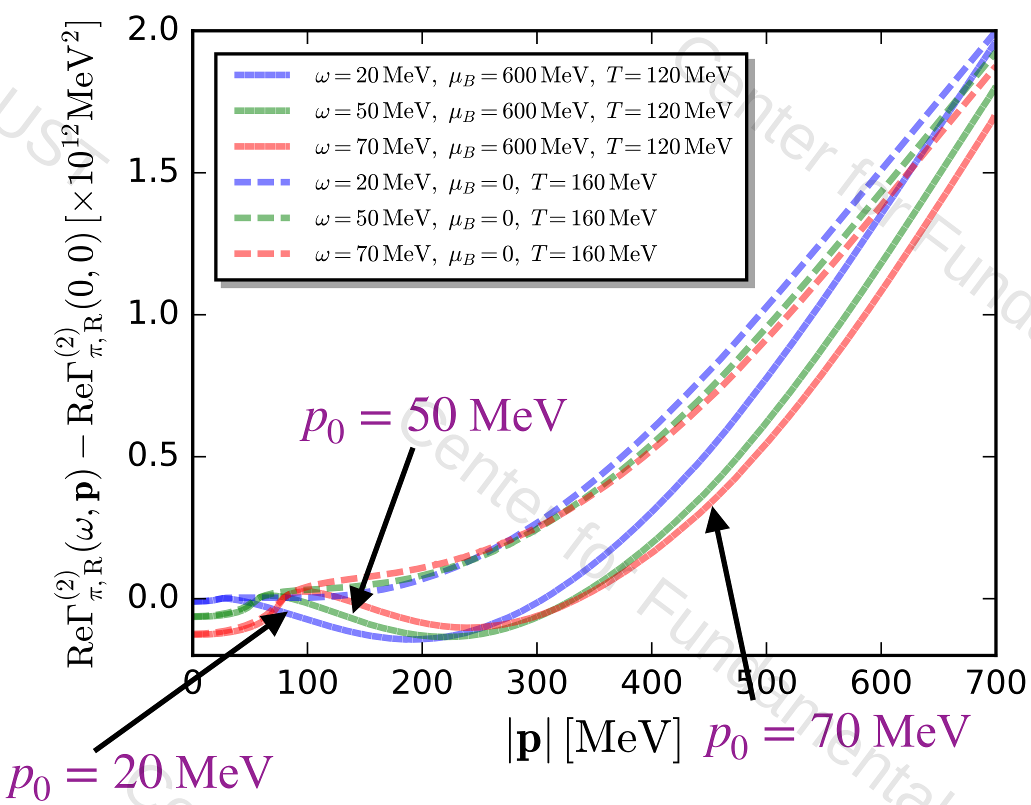
$$\Gamma^{(2)}_{\phi\phi,\mathrm{R}}(\omega,\mathbf{p}) = \lim_{\epsilon\to 0^+}\Gamma^{(2)}_{\phi\phi}\big(-\mathrm{i}(\omega+\mathrm{i}\epsilon),\mathbf{p}\big)$$

Note: not on data!

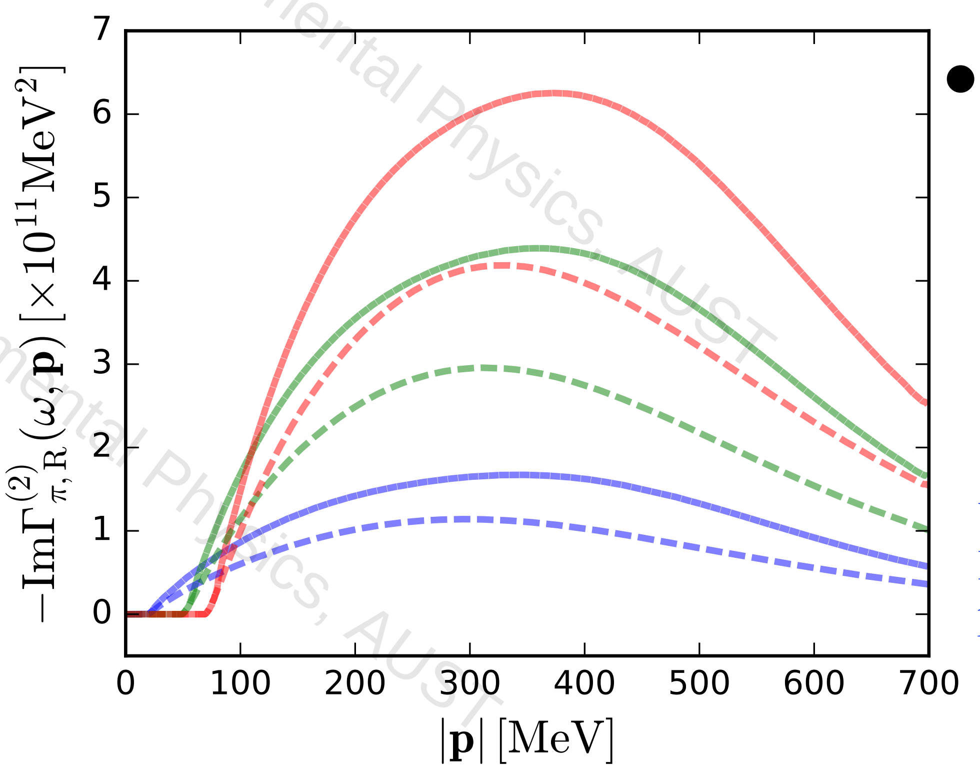


Spectral function

Real part of $\Gamma^{(2)}_{\phi\phi,\mathrm{R}}(p_0,\mathbf{p})$:



Imaginary part of $\Gamma^{(2)}_{\phi\phi,\mathrm{R}}(p_0,\mathbf{p})$:

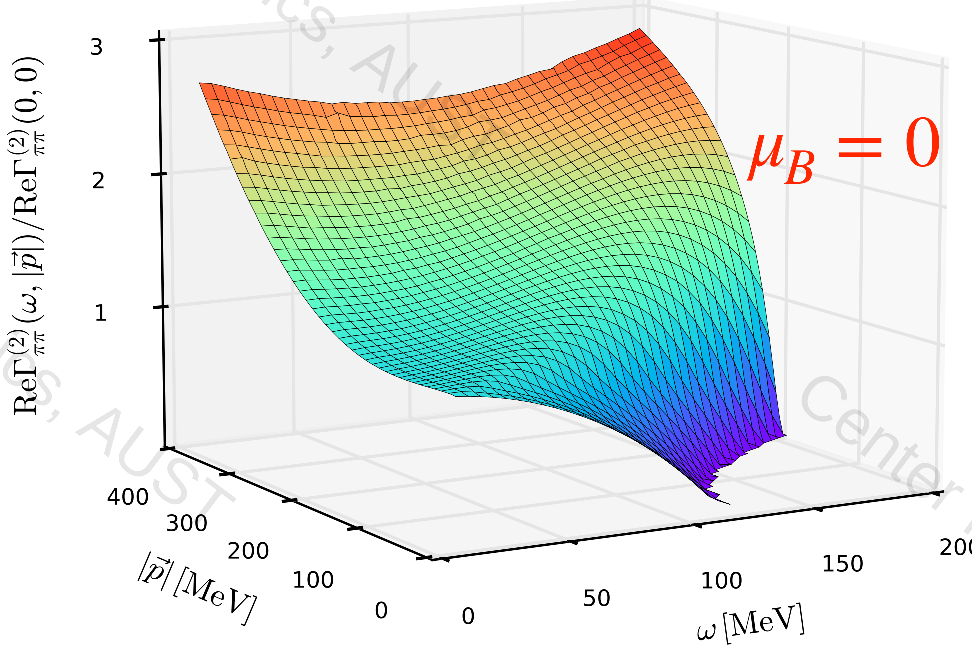


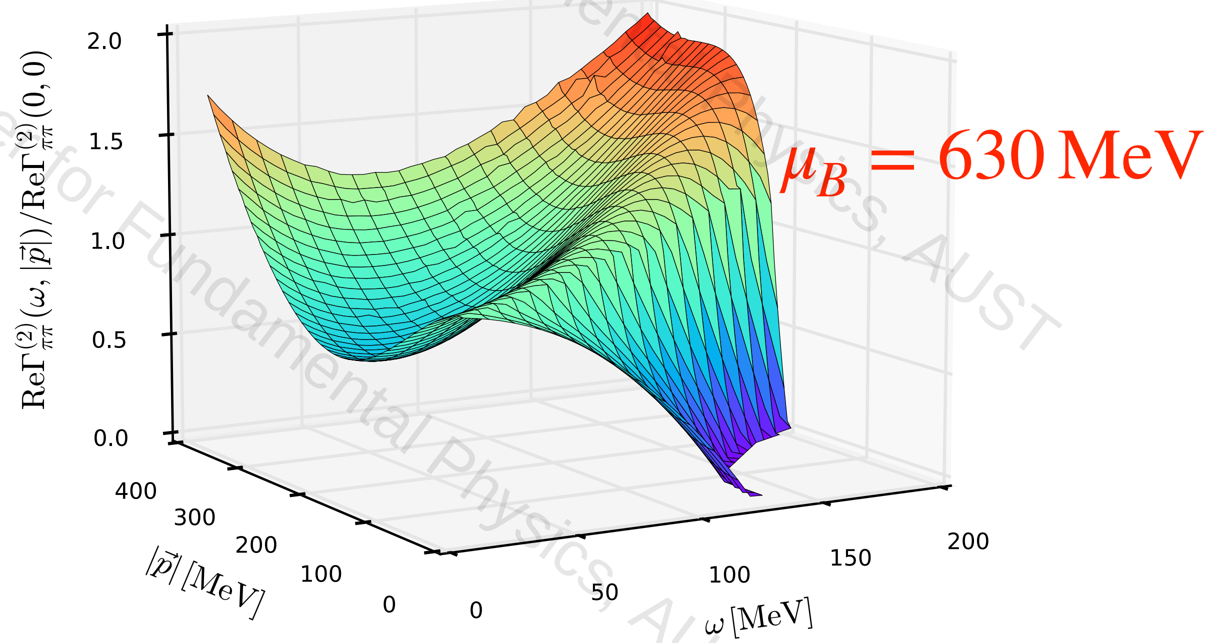
- Imaginary part of the mesonic two-point functions and spectral function are enhanced by the Landau damping effect

WF, Pawlowski, Pisarski, Rennecke, Wen, Yin, in preparation.

Real-time mesonic two-point functions

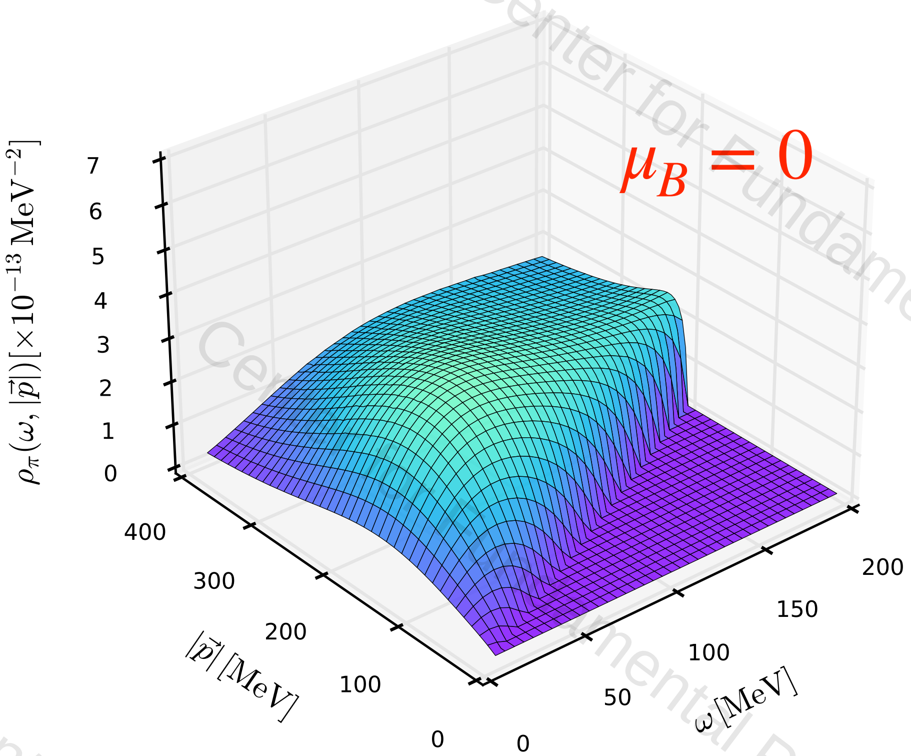
Real part:

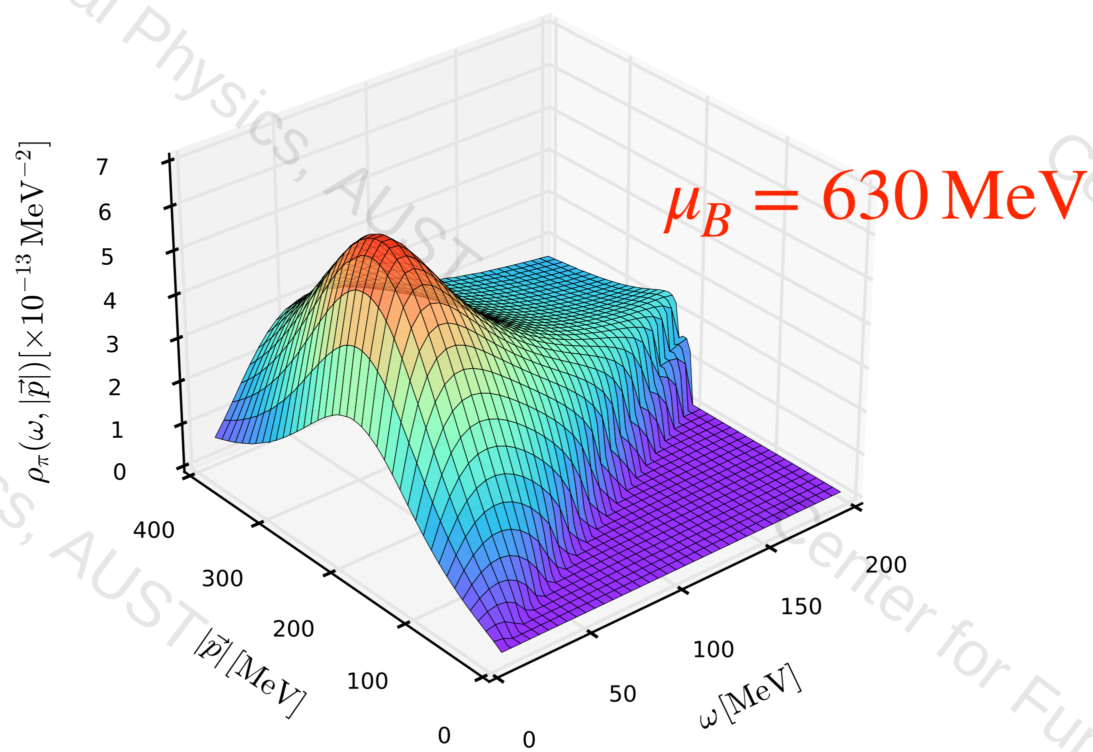




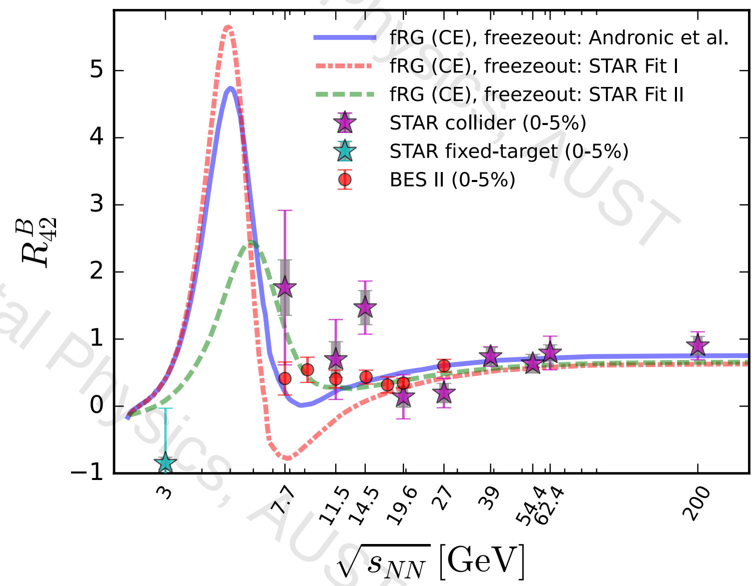
WF, Pawlowski, Pisarski, Rennecke, Wen, Yin, in preparation.

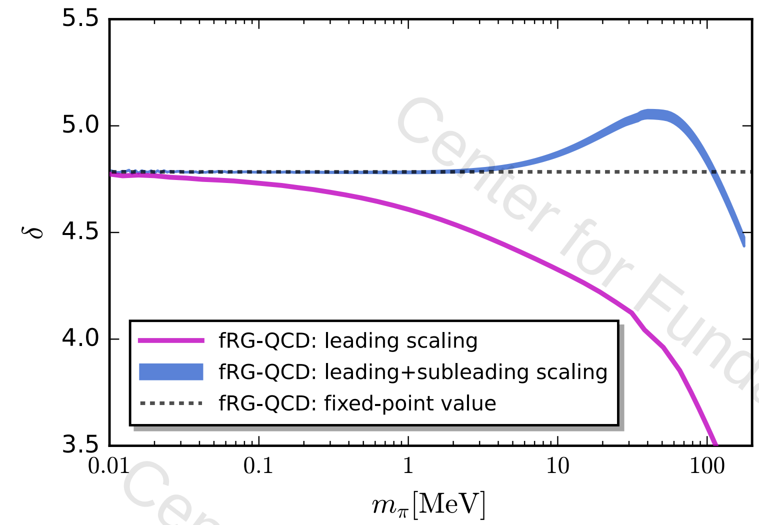
Spectral function:

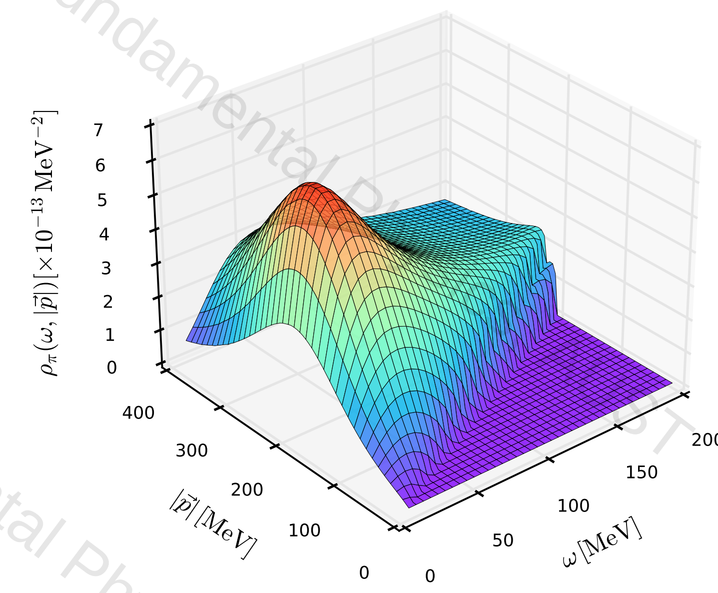




Summary





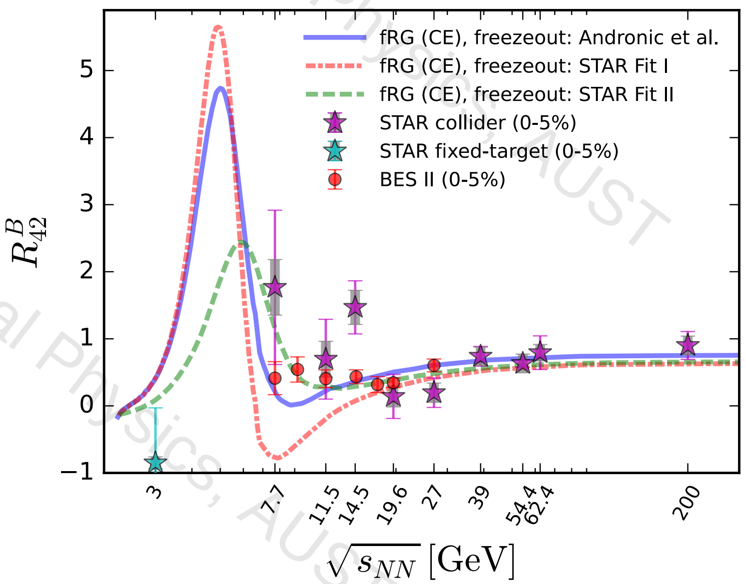


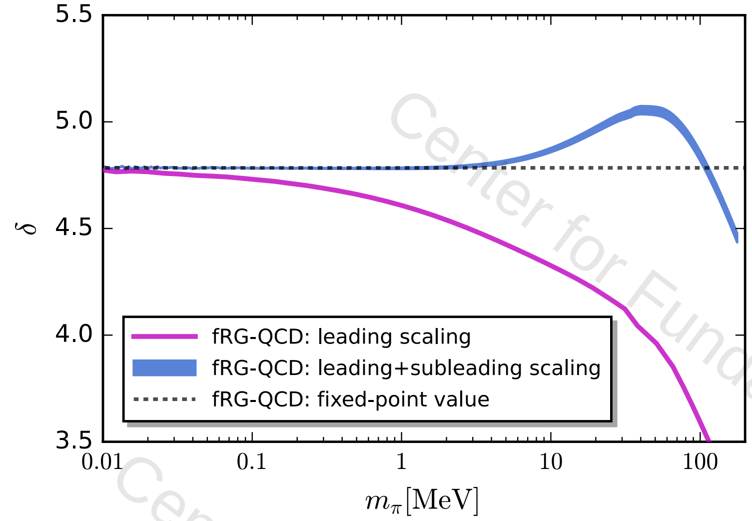
★ A prominent peak structure is found in baryon number fluctuations in the collision energy range of 3 GeV $\lesssim \sqrt{s_{\mathrm{NN}}} \lesssim$ 7.7 GeV.

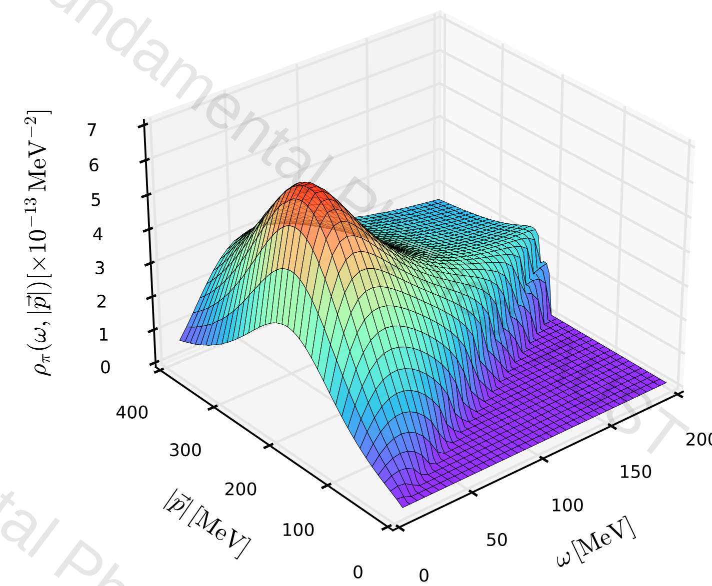
★ The size of the critical region in QCD is determined for the first time.

★ The moat regime is found to arise from the Landau damping.

Summary







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★ The size of the critical region in QCD is determined for the first time.

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Backup

QCD with dynamical hadronization

Introducing a RG scale dependent composite field:

$$\hat{\phi}_k(\hat{\varphi}), \text{ with } \hat{\varphi} = (\hat{A}, \hat{c}, \hat{\bar{c}}, \hat{q}, \hat{\bar{q}}),$$

Wetterich equation is modified as

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\big(G_k[\Phi]\,\partial_t R_k\big) + \mathrm{Tr}\left(G_{\phi\Phi_a}[\Phi]\frac{\delta\langle\partial_t\hat{\phi}_k\rangle}{\delta\Phi_a} R_\phi\right)$$

$$-\int\langle\partial_t\hat{\phi}_{k,i}\rangle\left(\frac{\delta\Gamma_k[\Phi]}{\delta\phi_i} + c_\sigma\delta_{i\,\sigma}\right),$$

WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

$$\langle\partial_t\hat{\phi}_k\rangle = \dot{A}_k\,\bar{q}\tau q + \dot{B}_k\,\phi + \dot{C}_k\,\hat{e}_\sigma,$$

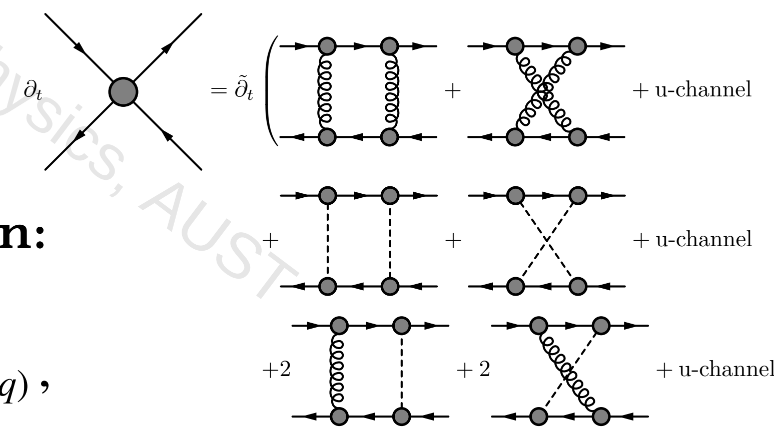
Gies, Wetterich, *PRD* 65 (2002) 065001; 69 (2004) 025001
Pawlowski, *AP* 322 (2007) 2831
Flörchinger, Wetterich, *PLB* 680 (2009) 371

Flow of four-quark couplings:

$$\partial_t\bar{\lambda}_q - 2\left(1+\eta_q\right)\bar{\lambda}_q - \bar{h}\,\dot{\bar{A}} = \overline{\mathbf{Flow}}^{(4)}_{(\bar{q}\tau q)(\bar{q}\tau q)},$$

choosing

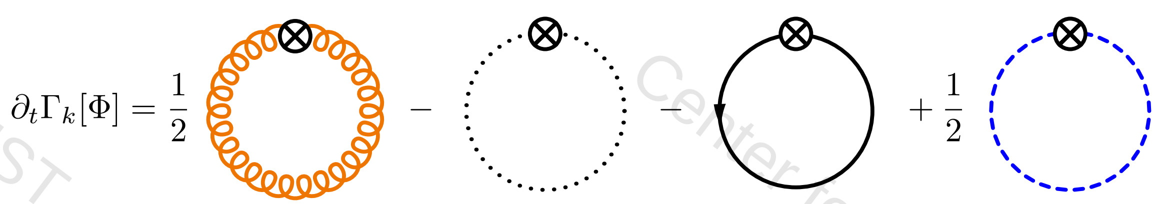
$$\bar{\lambda}_q \equiv 0, \qquad \forall k,$$

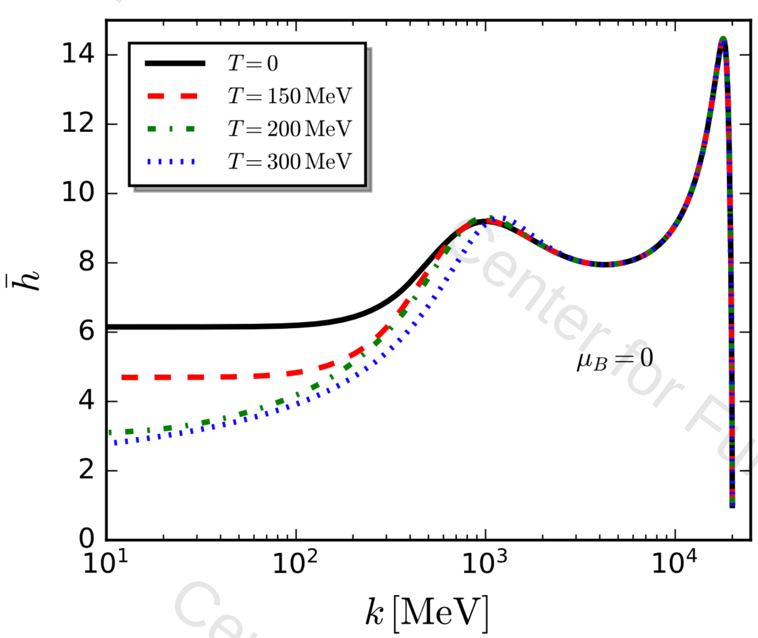


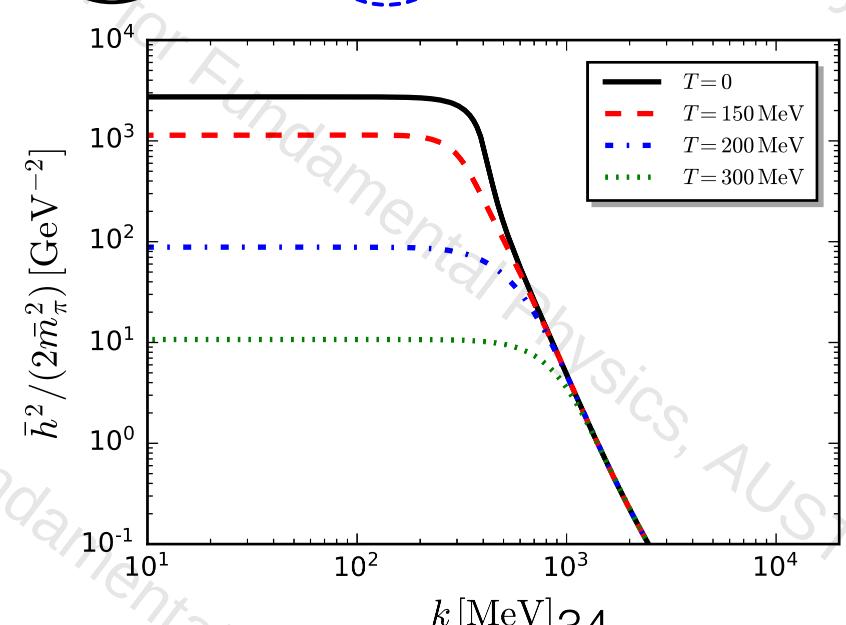
Hadronization function:

$$\dot{\bar{A}} = -\frac{1}{\bar{h}}\overline{\mathbf{Flow}}^{(4)}_{(\bar{q}\tau q)(\bar{q}\tau q)},$$

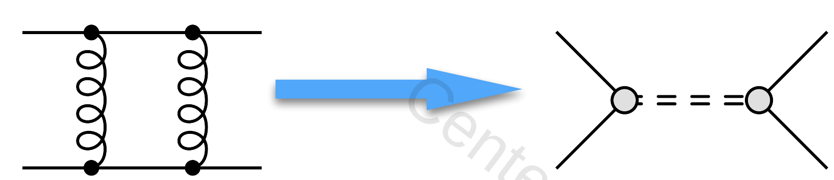
Flow equation:



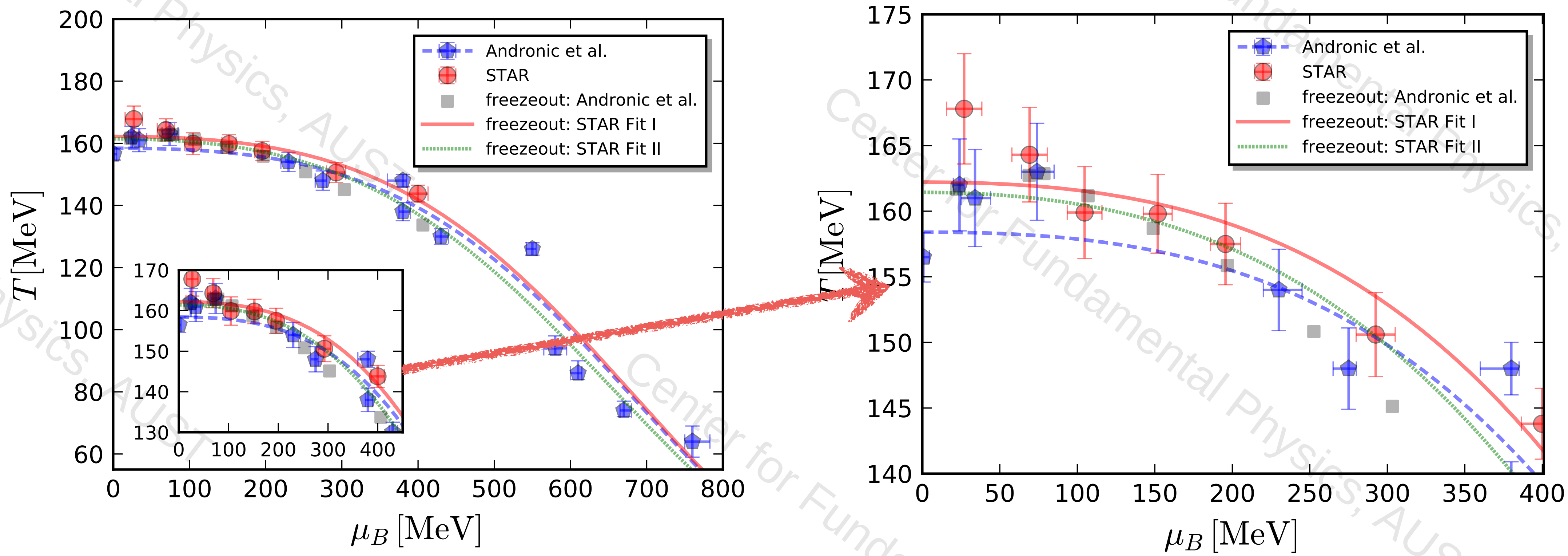




four-quark interaction encoded in Yukawa coupling:



Determination of the freeze-out curve



three freeze-out curves

1. freeze-out: Andronic *et al.*

Andronic, Braun-Munzinger, Redlich, *Nature* 561 (2018) 7723, 321

2. freeze-out: STAR Fit I

L. Adamczyk *et al.* (STAR), *PRC* 96 (2017), 044904

3. freeze-out: STAR Fit II

neglecting first two at low μ_B and the last one

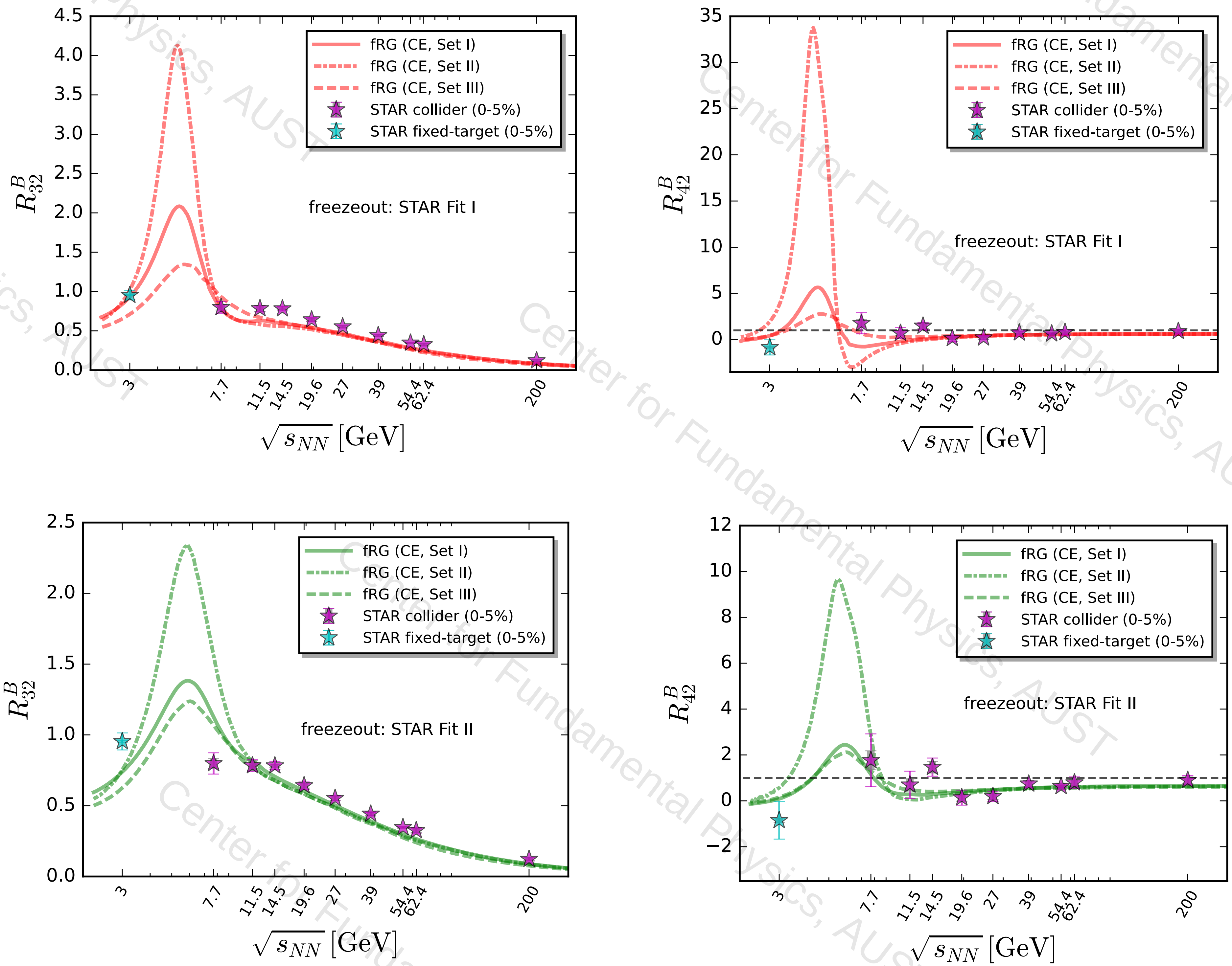
$$\mu_{B\,\mathrm{CF}} = \frac{a}{1 + 0.288\sqrt{s_{\mathrm{NN}}}},$$

$$T_{\mathrm{CF}} = \frac{T^{(0)}_{\mathrm{CF}}}{1 + \exp\left(2.60 - \ln(\sqrt{s_{\mathrm{NN}}})/0.45\right)}$$

all data points

- freeze-out curve should not rise with μ_B
- convexity of the freeze-out curve

Dependence of the location of CEP



STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301

fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508