

# Polarization Dynamics in Charged Magnetized Quark-Gluon Plasma

热烈祝贺安徽理工大学基础物理研究中心成立！

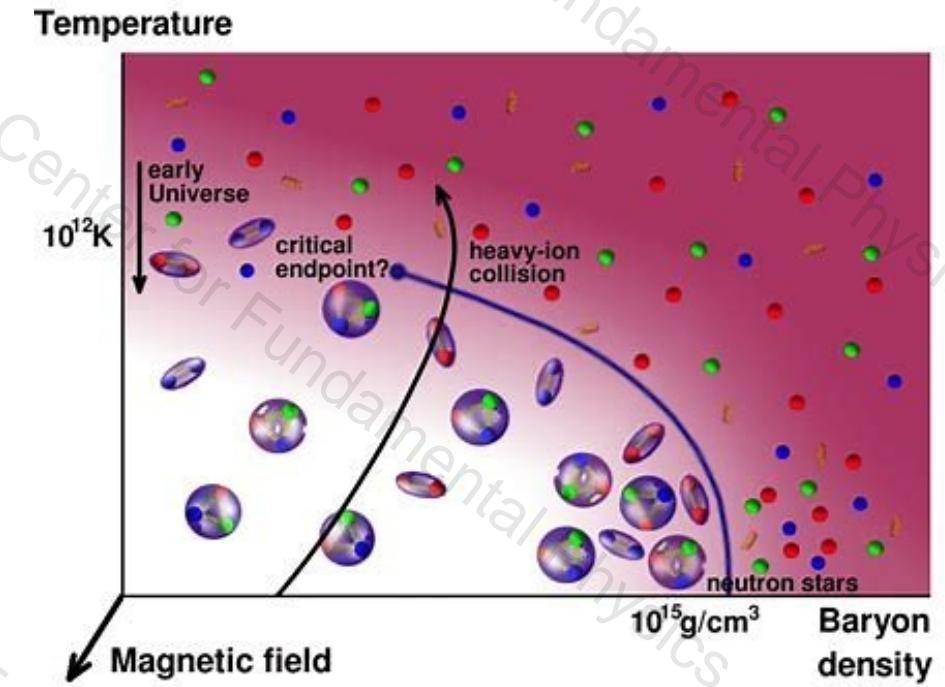
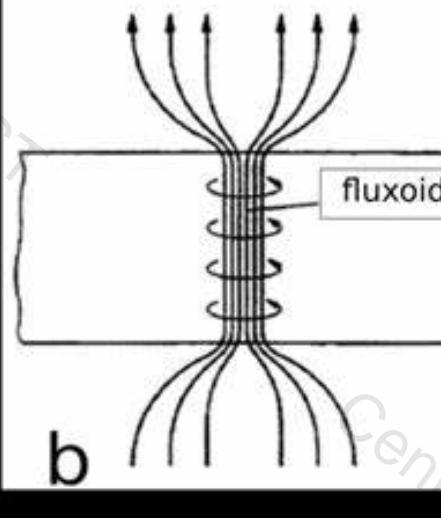
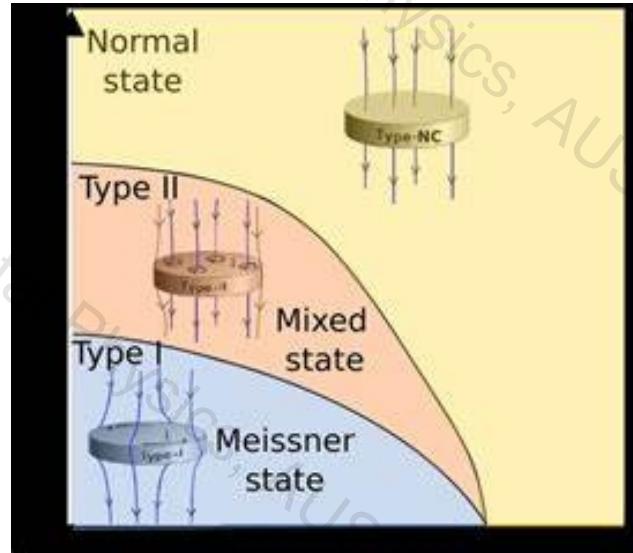


Shu Lin  
Sun Yat-Sen University

# Outline

- ◆ Magnetic field induced phases and transports
- ◆ Uncertainty in magnetic field in heavy ion collisions
- ◆ Magnetized QGP in HIC: a spinless fluid or a magnet?
- ◆ Dense magnetized QED matter: paramagnet
- ◆ Dense & hot magnetized QCD matter: paramagnet
- ◆ Polarization dynamics in HIC
- ◆ Conclusion and outlook

# Phases under magnetic field

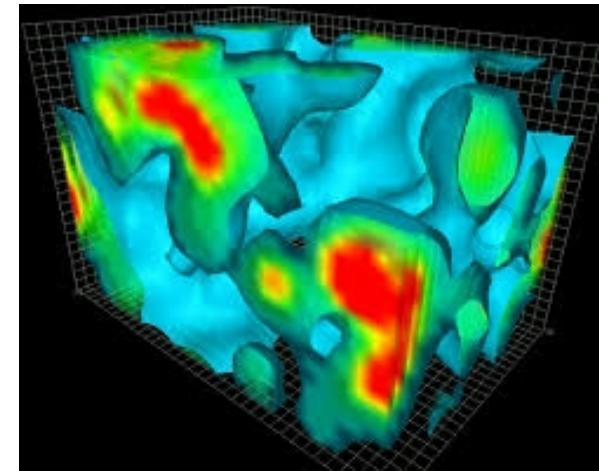


Superconductor under B

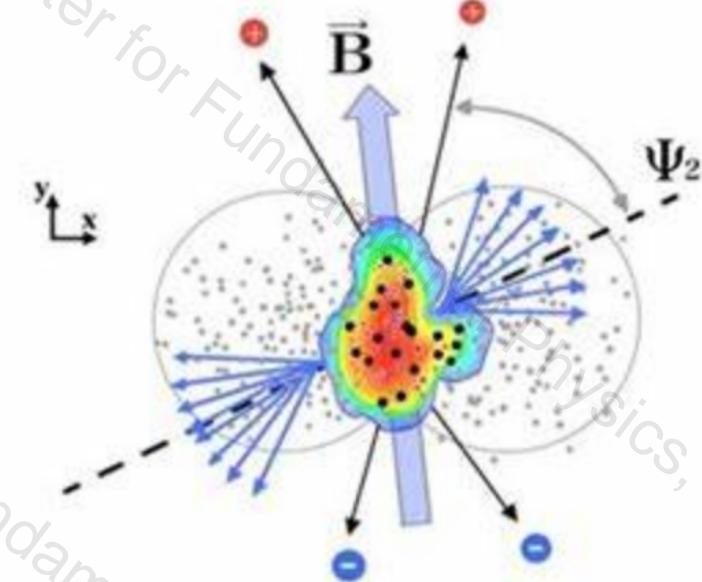
QCD phases under B

# Transport under magnetic field: chiral magnetic effect

Chiral magnetic effect  $\mathbf{J}_{\text{CME}} = \frac{e^2}{2\pi^2} \mu_5 \mathbf{B}$



Kharzeev, McLerran,  
Warringa, NPA 2008



$$\partial_\mu J_5^\mu = -\frac{g^2 N_f}{16\pi^2} \text{Tr} [\epsilon^{\mu\nu\rho\sigma} G_{\mu\nu} G_{\rho\sigma}]$$

CME measures topological fluctuation in quark-gluon plasma

# Transport under magnetic field: chiral magnetic effect

Chiral magnetic effect  $\mathbf{J}_{\text{CME}} = \frac{e^2}{2\pi^2} \mu_5 \mathbf{B}$

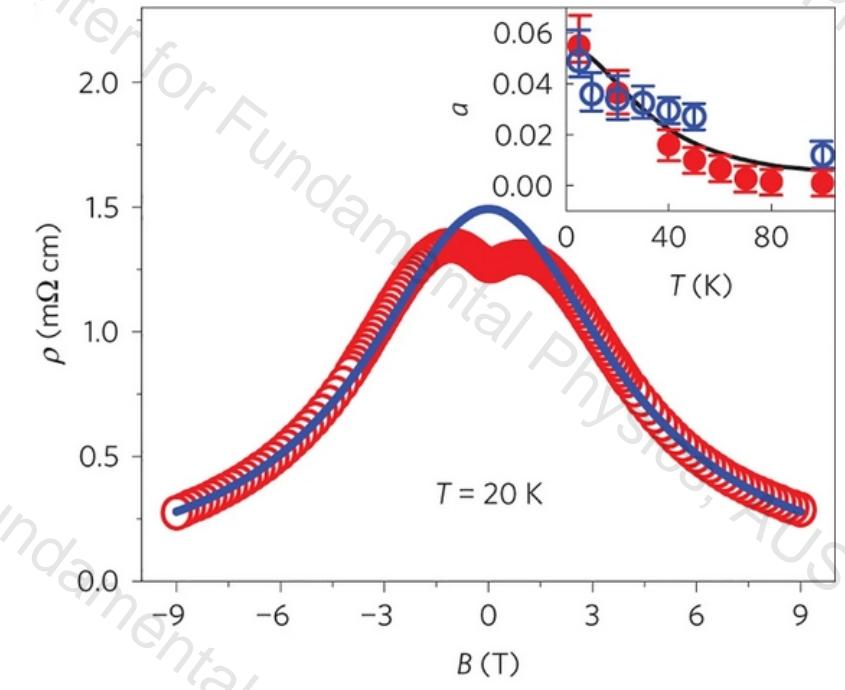
$$\partial_\mu J_5^\mu = -\frac{e^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \sim \mathbf{E} \cdot \mathbf{B}$$



$$\sigma_{\text{CME}} = \sigma_0 + a(T)B^2,$$

Negative magnetoresistance  
in ZrTe5

Li, Kharzeev, et al, Nature physics 2016



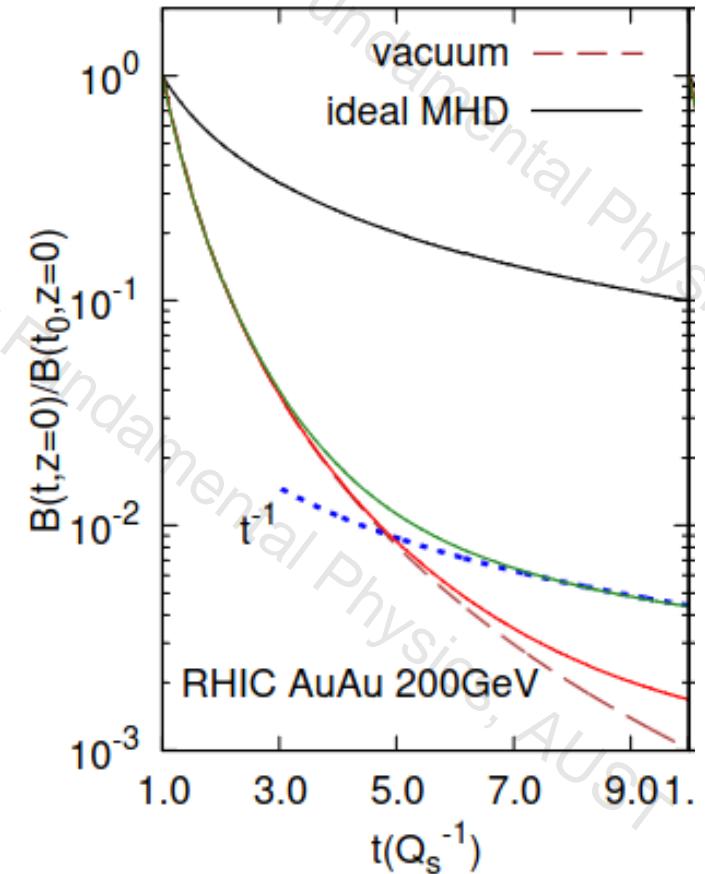
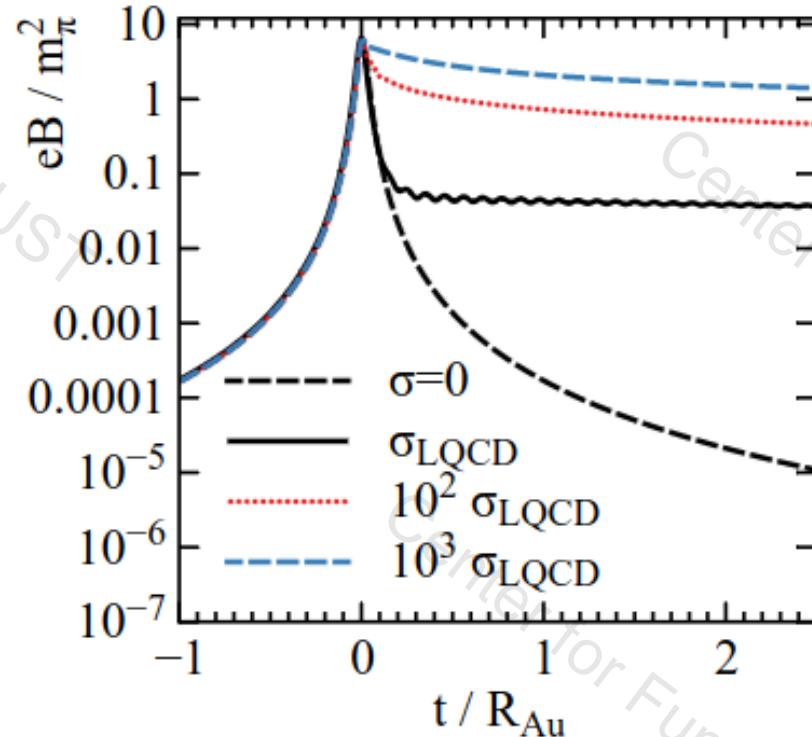
# Magnetic field in Heavy ion Collisions

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

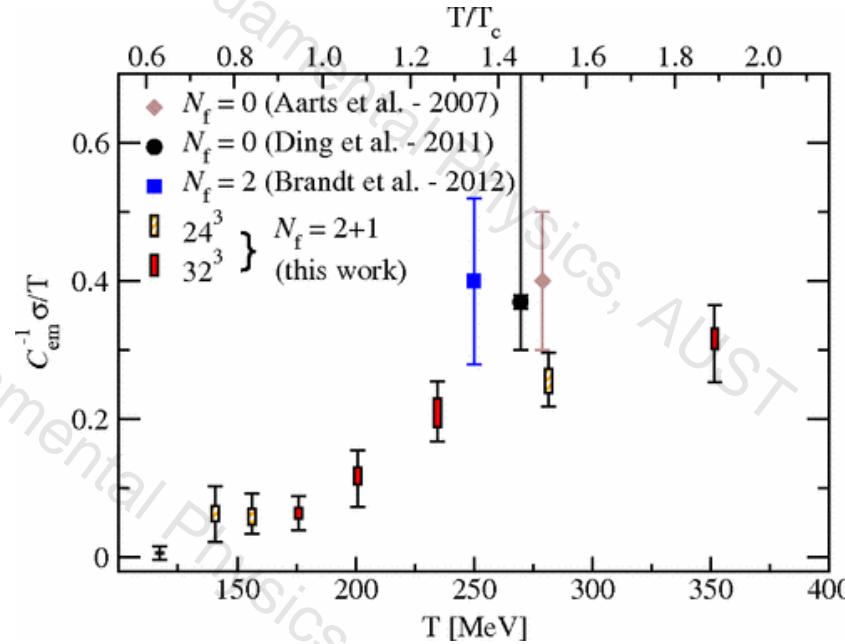
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$



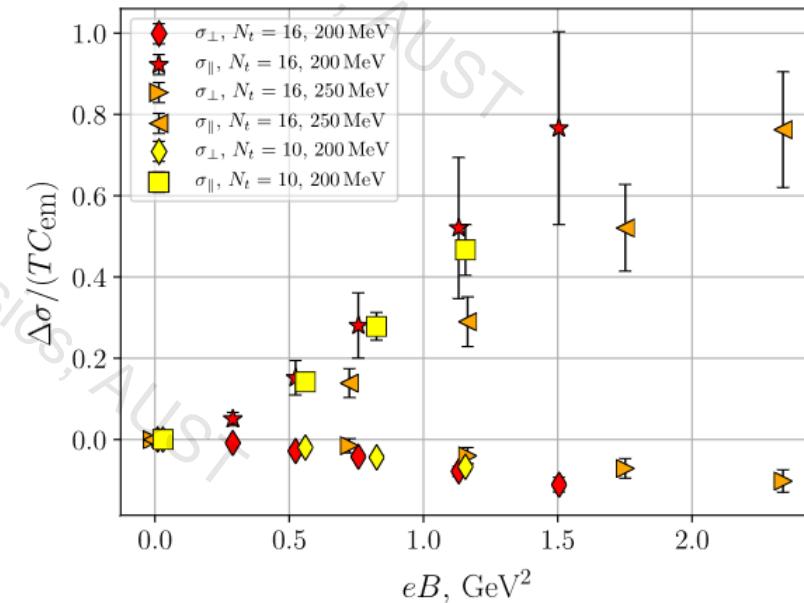
Expect short life time of B

Skokov, McLerran,  
NPA 2014  
Yan, Huang, PRD  
2023

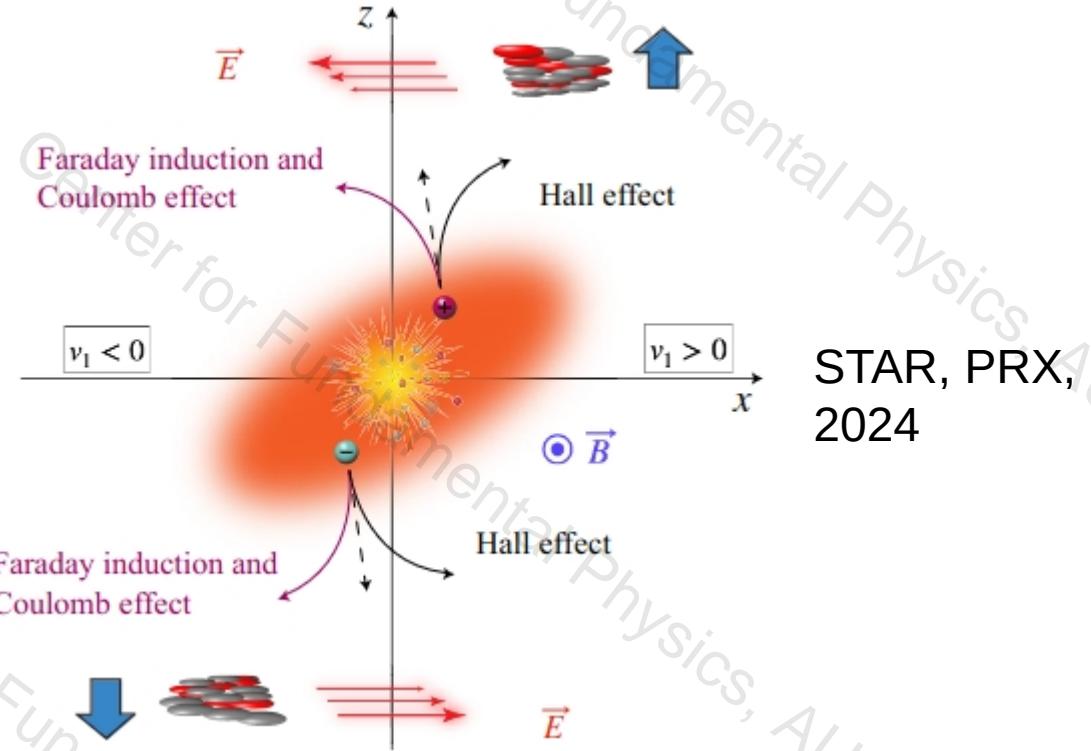
# Electric conductivity from Lattice & Experiment



Aarts et al,  
2007, 2013  
Ding et al,  
2011, 2014



Kotov et al,  
PRD, 2020



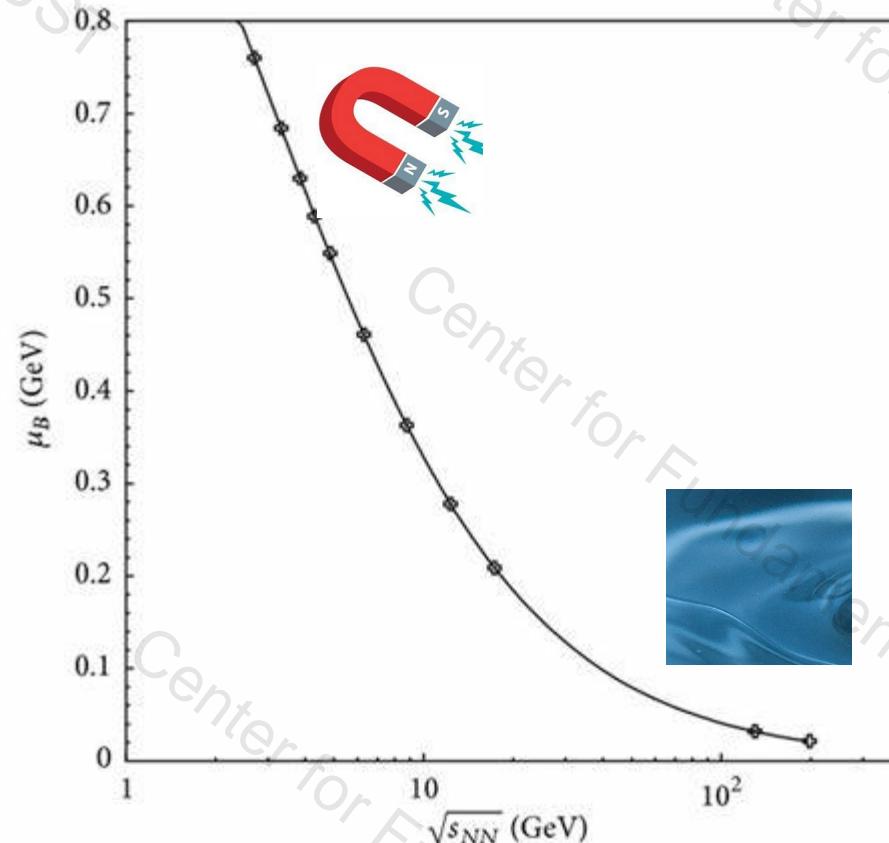
Lattice disfavors large conductivity  
Experiment measurement of  $v_1$   
consistent with lattice

STAR, PRX,  
2024

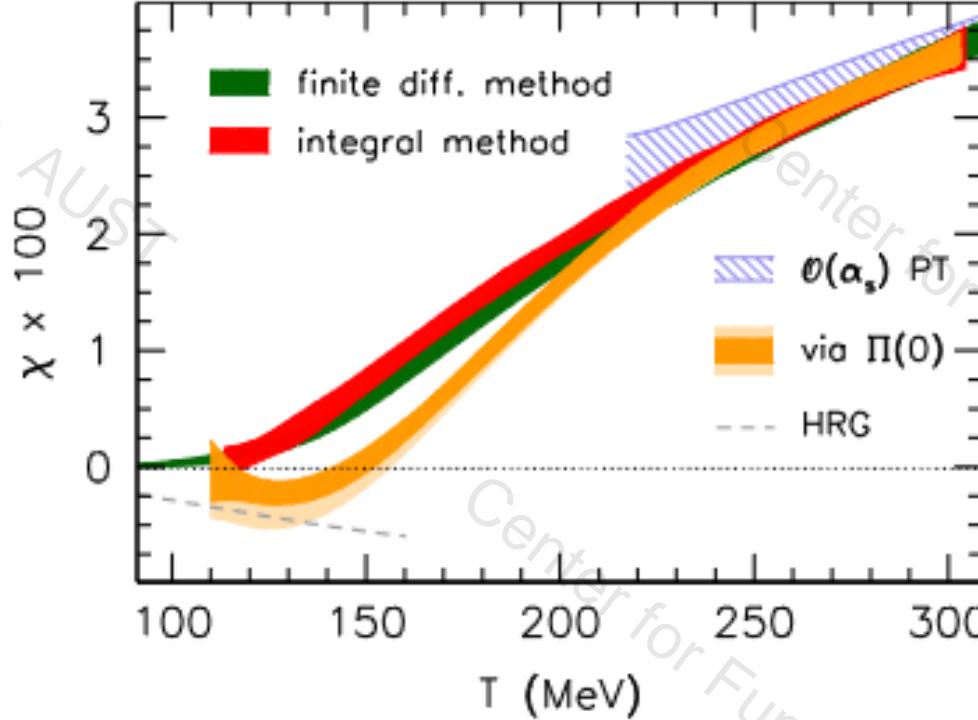
# What can QGP be other than conducting medium?

0<sup>th</sup> approx: spinless conducting fluid

1<sup>st</sup> approx: consisting of spinning particles,  
magnetized by B-field, like magnet



# QGP as a paramagnet (weak B)



Bali, Endrodi, Piemonte,  
JHEP 2020

$$\chi^{\text{OAM}} = -\frac{1}{3}\chi^{\text{spin}}$$

# QGP as a paramagnet (strong B)

$$E = \sqrt{m^2 + p_3^2 + 2neB}$$

$n = 0$  lowest Landau level dominated

positive charge     $s_z = 1/2$

negative charge     $s_z = -1/2$

magnetization from spin of LLL states

Sheng, Rischke, Vasak,  
Wang, EPJA 2017  
Gorbar, Miransky,  
Sovkovy, Sukhachov,  
JHEP 2017  
SL, Yang, JHEP 2021

# What if B turned off (strong B)?

turned off adiabatically: LLL states demagnetized

$$\tau_B \gg \tau_{rel}$$

turned off suddenly: free relaxation of LLL states

$$\tau_B \ll \tau_{rel}$$

momentum isotropization

$$\tau_{rel} \sim \frac{1}{g^4 T}$$

It is likely QGP remains a LLL state after rapid decay of B

# Photon self-energy in massless QED under strong B

soft photon

symmetric

anti-symmetric

$$\Pi_R^{\mu\nu} = -\frac{e^3 B}{2\pi^2} \frac{q_3^2 u^\mu u^\nu + q_0^2 b^\mu b^\nu + q_0 q_3 u^{\{\mu} b^{\nu\}}}{(q_0 + i\epsilon)^2 - q_3^2} + \frac{ie^2 \mu}{2\pi^2} \left( q_0 \epsilon^{\mu\nu\rho\sigma} + u^{[\mu} \epsilon^{\nu]\lambda\rho\sigma} q_\lambda^T \right) u_\rho b_\sigma.$$

Chiral Magnetic Wave in  
lowest Landau level approx

Kharzeev, Yee, PRD 2011  
Fukushima PRD, 2011  
Gao, Mo, SL, PRD 2020

Hall effect: drift velocity +  
charge density  $\rightarrow$  current

Hidaka, Fukushima, JHEP 2020  
SL, Yang, JHEP 2021  
Yang, PRD 2022

$u^\mu = (1, 0, 0, 0)$  fluid

$b^\mu = (0, 0, 0, 1)$  B-field

$q^\mu$  photon momentum

# Photon dispersions in massless QED under strong B

$$(\partial^2 \eta^{\mu\nu} - \partial^\mu \partial^\nu) A_{\nu,r} = j_r^\mu = -i \int d^4y \Pi_{ar}^{\mu\nu}(x, y) A_{\nu,i}$$

$$q_0^2 = \tilde{B} + q^2$$

gapped mode

low-energy modes

$$q_0^2 = \frac{1}{2} \left( \tilde{\mu}^2 + q_\perp^2 + 2q_3^2 - \sqrt{4\tilde{\mu}^2 q_3^2 + (q_\perp^2 + \tilde{\mu}^2)^2} \right) \equiv x_1^2,$$

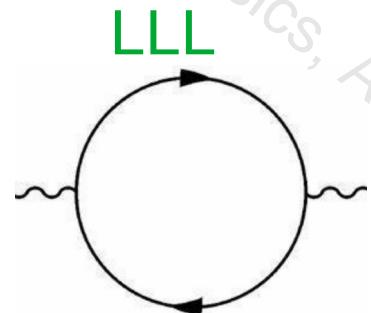
$$q_0^2 = \frac{1}{2} \left( \tilde{\mu}^2 + q_\perp^2 + 2q_3^2 + \sqrt{4\tilde{\mu}^2 q_3^2 + (q_\perp^2 + \tilde{\mu}^2)^2} \right) \equiv x_2^2.$$

$$eB \gg \mu q$$

$$\mu = 0$$

$$q_0^2 = q_3^2$$

$$q_0^2 = q^2 \text{ free}$$



$$\tilde{\mu} = e^2 \mu / 2\pi^2$$

$$\tilde{B} = e^3 B / 2\pi^2$$

$$q_\perp^2 = q_1^2 + q_2^2$$

medium Fermi liquid-like rather than fluid-like

# Photon polarization in massless QED under strong B

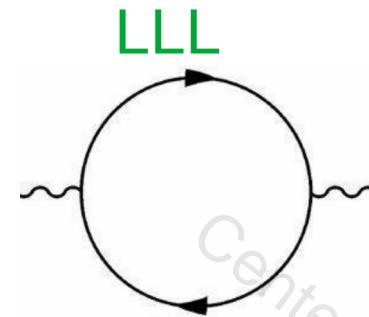
Hall dynamics requires

$$q_0, q \sim \tau_R^{-1} \sim e^4 \mu, \quad \text{orange arrow} \quad \tilde{\mu} \sim e^2 \mu \gg q$$

$$q_0^2 = x_1^2 : \frac{A_1}{A_0} = \frac{i(q_2 q + i q_1 |q_3|)}{q_\perp^2 q} \tilde{\mu}, \quad \frac{A_2}{A_0} = -\frac{i(q_1 q - i q_2 |q_3|)}{q_\perp^2 q} \tilde{\mu}, \quad \frac{A_3}{A_0} = \frac{q_3}{|q_3| q} \tilde{\mu}.$$

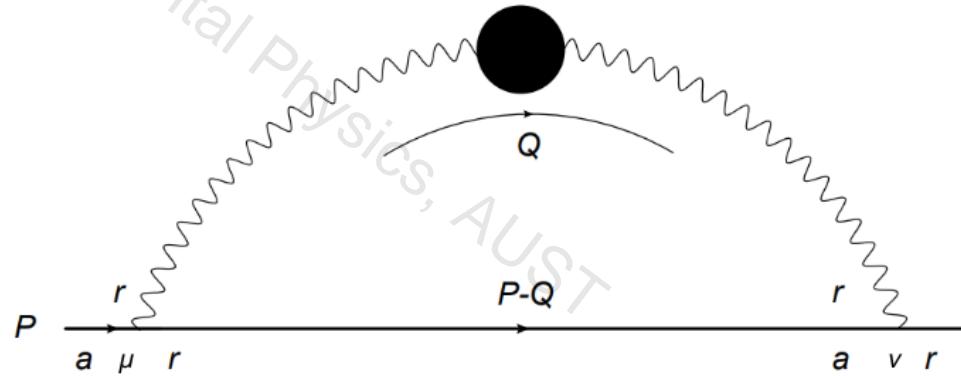
simple interpretation at  $q_3 \gg q_\perp$

$$\frac{A_1}{A_2} \simeq -i$$



One photon polarization favored due  
to interaction with spin polarized  
CMW in charged magnetized medium

# Self-energy of unpolarized massless probe fermion



$$D_{\mu\nu}^{rr}(Q) = -2i\pi \epsilon(q_0) (S_{\mu\nu}(Q) + A_{\mu\nu}(Q)\tilde{\mu}) \left( \frac{1}{2} + f_\gamma(q_0) \right) \left( \frac{\delta(q_0^2 - x_1^2)}{q_0^2 - x_2^2} + \frac{\delta(q_0^2 - x_2^2)}{q_0^2 - x_1^2} \right)$$

symmetric   anti-symmetric

$$S_{ra(0)}(P) = \frac{iP}{(p_0 + i\epsilon)^2 - p^2}$$

$$\Gamma_L \simeq \frac{c_3 p_3}{p} = \frac{e^2 T q_{UV}^2}{8\pi \tilde{\mu} p} \epsilon(p_3),$$

$$\Gamma_R \simeq -\frac{c_3 p_3}{p} = -\frac{e^2 T q_{UV}^2}{8\pi \tilde{\mu} p} \epsilon(p_3),$$

anti-symmetric component leads  
to splitting in damping rate

# Implication for polarization dynamics

$$\Gamma_L \simeq \frac{c_3 p_3}{p} = \frac{e^2 T q_{\text{UV}}^2}{8\pi \tilde{\mu} p} \epsilon(p_3),$$

$$\Gamma_R \simeq -\frac{c_3 p_3}{p} = -\frac{e^2 T q_{\text{UV}}^2}{8\pi \tilde{\mu} p} \epsilon(p_3).$$

amplified modes:

right-handed  $p_3 > 0$

left-handed  $p_3 < 0$ .

Positive spin polarization along B

charged magnetized QED medium behaves like paramagnet  
in dynamical sense

# Gluon self-energy in massless QCD under strong B soft gluon

CMW in LLL approx

$$\Pi_R^{\mu\nu,AB} = \left[ -\frac{g^2 e B}{2\pi^2} \frac{q_3^2 u^\mu u^\nu + q_0^2 b^\mu b^\nu + q_0 q_3 u^{\{\mu} b^{\nu\}}}{(q_0 + i\epsilon)^2 - q_3^2} + \frac{ig^2}{2\pi^2} \frac{\mu}{2} \left( q_0 \epsilon^{\mu\nu\rho\sigma} + u^{[\mu} \epsilon^{\nu]\lambda\rho\sigma} q_\lambda^T \right) u_\rho b_\sigma \right. \\ \left. - P_T^{\mu\nu} \Pi_T - P_L^{\mu\nu} \Pi_L \right] \delta^{AB},$$

chromo-Hall effect: balance between chrome-electric force and Lorentz force

gluon self-interaction

$$u^\mu = (1, 0, 0, 0) \quad \text{fluid}$$

$$b^\mu = (0, 0, 0, 1) \quad \text{B-field}$$

$$q^\mu \cdot \text{gluon momentum}$$

$$\Pi_T = m^2 (x^2 + (1-x^2)xQ_0(x)),$$

$$\Pi_L = -2m^2(x^2-1)(1-xQ_0(x)),$$

$$m^2 = \frac{1}{6} N_c g^2 T^2$$

# Two limits of QGP medium

$$\Pi_R^{\mu\nu,AB} = \left[ -\frac{g^2 e B}{2\pi^2} \frac{q_3^2 u^\mu u^\nu + q_0^2 b^\mu b^\nu + q_0 q_3 u^{\{\mu} b^{\nu\}}}{(q_0 + i\epsilon)^2 - q_3^2} + \frac{i g^2}{2\pi^2} \frac{\mu}{2} \left( q_0 \epsilon^{\mu\nu\rho\sigma} + u^{[\mu} \epsilon^{\nu]\lambda\rho\sigma} q_\lambda^T \right) u_\rho b_\sigma \right. \\ \left. - P_T^{\mu\nu} \Pi_T - P_L^{\mu\nu} \Pi_L \right] \delta^{AB},$$

density dominate

$$\bar{\mu}^2 \gg \Pi_{T/L}$$

medium like Fermi-liquid

temperature dominate

$$\bar{\mu}^2 \ll \Pi_{T/L}$$

medium like fluid

$$\bar{\mu} \sim g^2 \mu, \quad \Pi_{T/L} \sim g^2 T^2$$

# Density dominated limit

$$D_{\mu\nu}^{rr,A}(Q) = -2i\pi \epsilon(q_0) \left( \frac{1}{2} + f_g(q_0) \right) \left( \frac{\delta(q_0^2 - \bar{x}_1^2)}{q_0^2 - \bar{x}_2^2} + \frac{\delta(q_0^2 - \bar{x}_2^2)}{q_0^2 - \bar{x}_1^2} \right) A_{\mu\nu}(Q) \bar{\mu}.$$

similar to QED case

$$q_0, q \sim \tau_R^{-1} \sim g^4 \mu \ll \bar{\mu}.$$

damping from scattering  
with CMW states

$$\Gamma_L \simeq \frac{N_c^2 - 1}{2N_c} \frac{g^2 T q_{\text{UV}}^2}{8\pi \bar{\mu} p} \epsilon(p_3),$$

$$\Gamma_R \simeq -\frac{N_c^2 - 1}{2N_c} \frac{g^2 T q_{\text{UV}}^2}{8\pi \bar{\mu} p} \epsilon(p_3).$$

charged magnetized QGP in density  
dominated limit behaves like paramagnet  
in dynamical sense

# Temperature dominated limit

$$D_{\mu\nu}^{rr,A}(Q) = 2i\text{Im} \left[ \frac{Q^2 q^2}{(Q^2 - \Pi_T) (q^2 Q^2 (q_0^3 - q_3^2) - Q^2 q_3^2 \Pi_T - q_0^2 q_\perp^2 \Pi_L)} \right] \left( \frac{1}{2} + f_g(q_0) \right) A_{\mu\nu} \bar{\mu}.$$

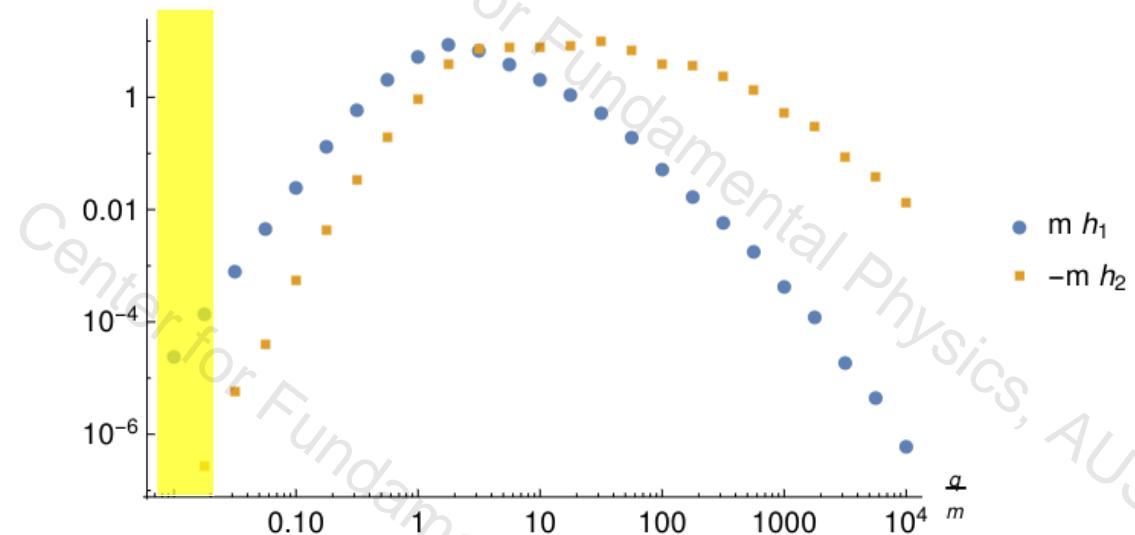
$$q_0, q \sim \tau_R^{-1} \sim g^4 T.$$

damping from scattering with gluons

$$\Gamma_L \simeq -\epsilon(p_3) \left( H_2 - \frac{|p_3| H_1}{p} \right),$$

$$\Gamma_R \simeq \epsilon(p_3) \left( H_2 - \frac{|p_3| H_1}{p} \right).$$

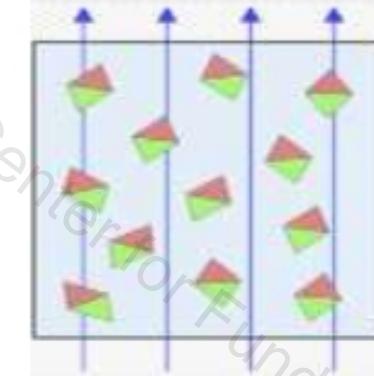
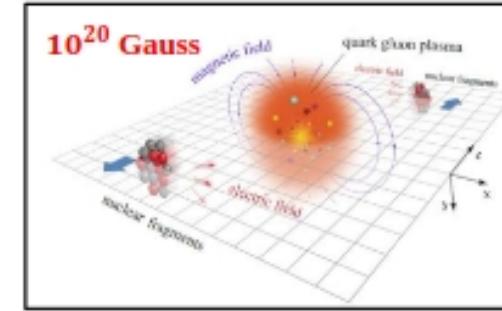
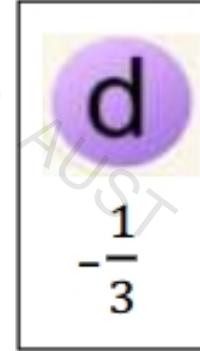
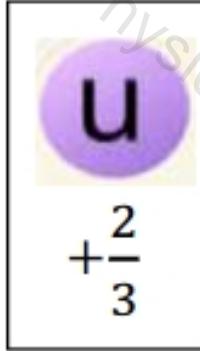
$$H_i \sim \int dq h_i \quad \xrightarrow{\text{blue arrow}} \quad H_1 \gg H_2$$



charged magnetized QGP in temperature  
dominated limit behaves like paramagnet  
in dynamical sense

L. Dong, SL, 2403.12615

# Implication for polarization dynamics in HIC



- ♦ Low energy HIC produces medium with baryonic and electric charge
- ♦ Initial magnetic field decays quickly and magnetizes QGP
- ♦ Magnetized QGP continues to polarize quarks produced at later stage like strange quark, effectively extend life time of B

# Conclusion

- ◆ Splitting of damping rate of spin component of probe fermion in charged magnetized QED matter
- ◆ Splitting of damping rate of spin component of probe quark in charged magnetized QCD matter
  - ◆ Paramagnet charged QGP can polarize probe quark

# Outlook

- ◆ Beyond strong B field limit
- ◆ Beyond probe limit, backreaction to paramagnet

**Thank you!**